

定量生物に効く数値計算

2012年1月7日

慶應義塾大学 舟橋 啓



Keio University
1858
CALAMVS
GLADIO
FORTIOR

Windowsの人

<http://padre.perlide.org>

The screenshot shows a web browser window displaying the official website for Padre, the Perl IDE. The title bar reads "Padre, the Perl IDE". The address bar shows the URL "padre.perlide.org". The main content area features a large blue butterfly logo on the right. The text on the left describes Padre as a Perl IDE, suitable for both new Perl programmers and advanced developers. It highlights its extensible plugin system and support for multi-lingual and multi-technology projects. A "Download" button with a blue arrow icon is prominently displayed, and it is circled in red. Below the download button, a section titled "Features" lists several key capabilities: customizable syntax highlighting, syntax checking for Perl 5 and Perl 6, refactoring tools, context-sensitive help, code completion, and being beginner-friendly.

Padre, the Perl IDE

Perl Application Development and Refactoring Environment

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Padre, the Perl IDE

Padre is a Perl IDE, an integrated development environment, or in other words a **text editor** that is simple to use for new Perl programmers but also supports large multi-lingual and multi-technology projects.

Our primary focus is to create a peerless environment for learning Perl and creating Perl scripts, modules and distributions, with an extensible plug-in system to support the addition of related functionality and languages and to support advanced developers taking the editor anywhere they want it to go.

Download

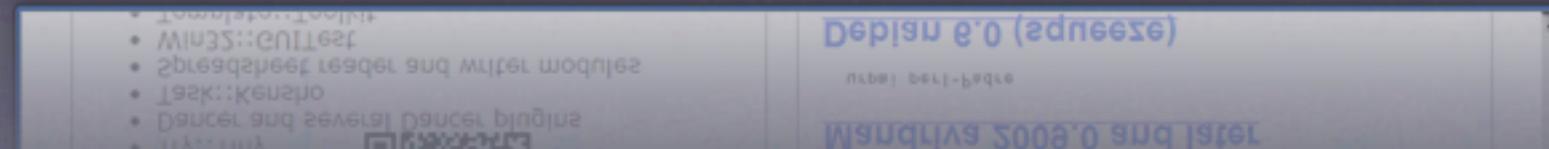
Features

- Customizable [syntax highlighting](#) for many languages and [visual editor effects](#)
- [Syntax checking for Perl 5 and Perl 6](#)
- Refactoring tools for [Perl 5](#) and [Perl 6](#)
- Context sensitive help and [code completion](#)
- Beginner-friendly

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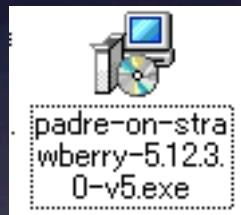
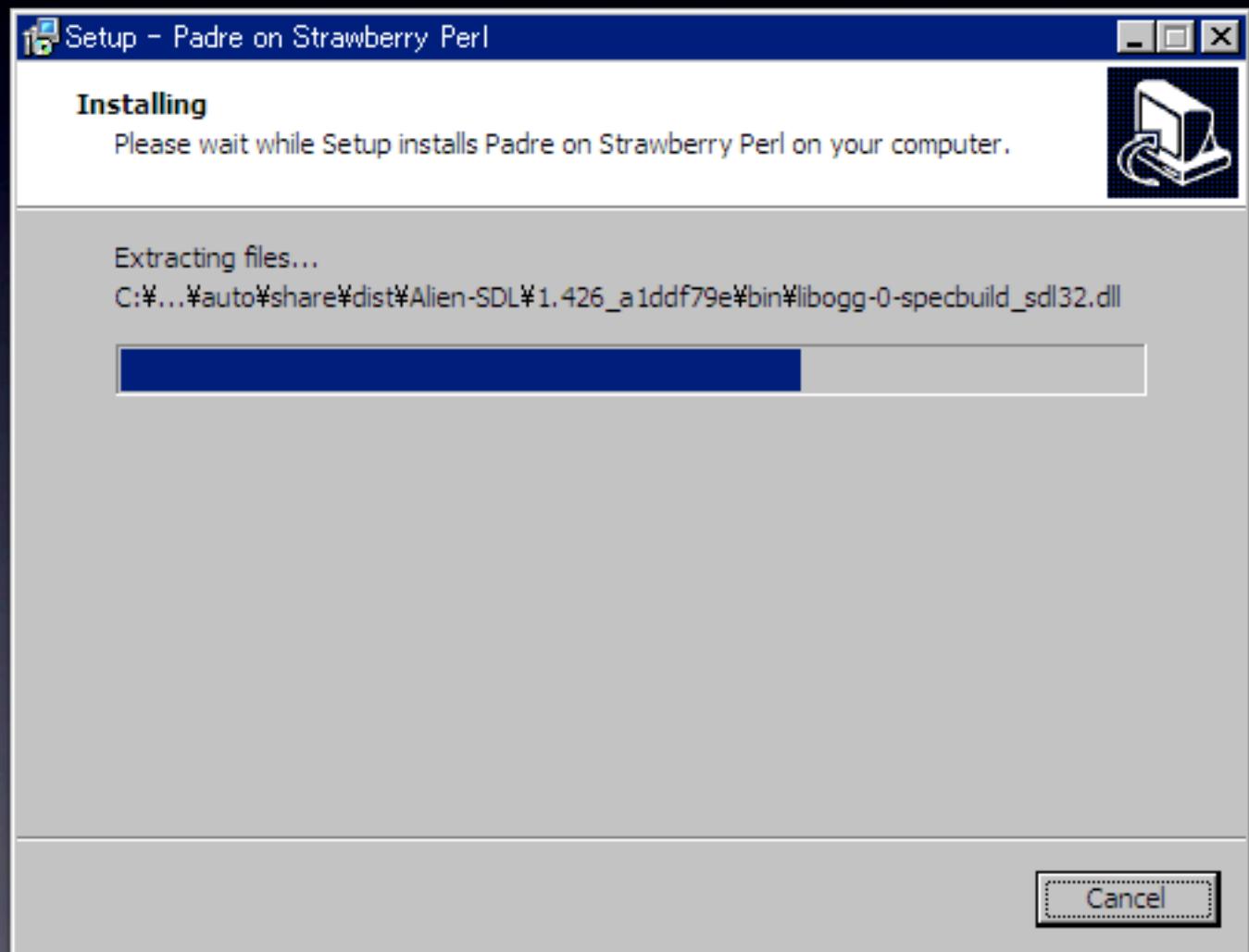
Padreをダウンロード

<http://padre.perlide.org>



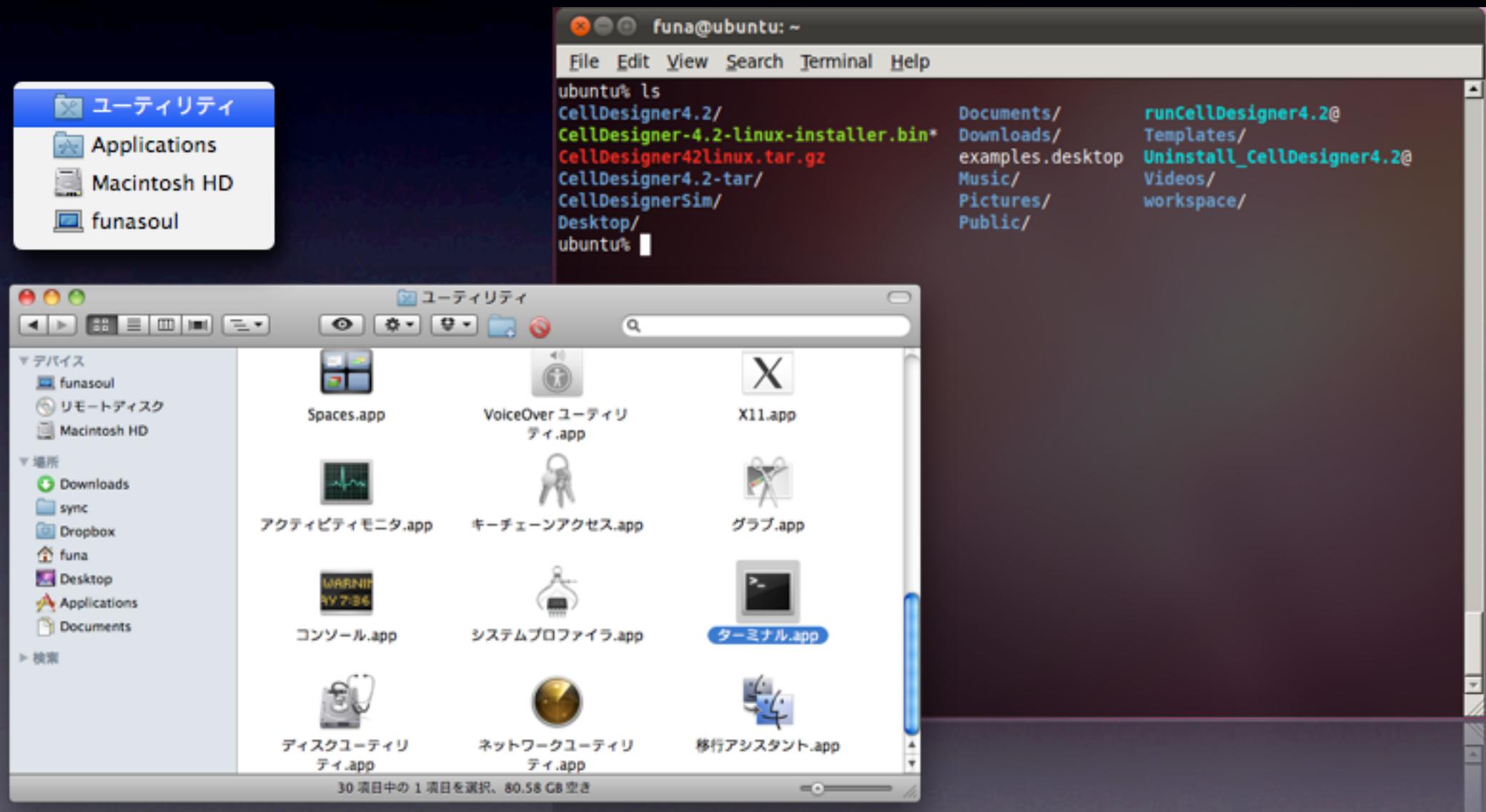
Padreをinstall

<http://padre.perlide.org>



Mac, Linuxの人

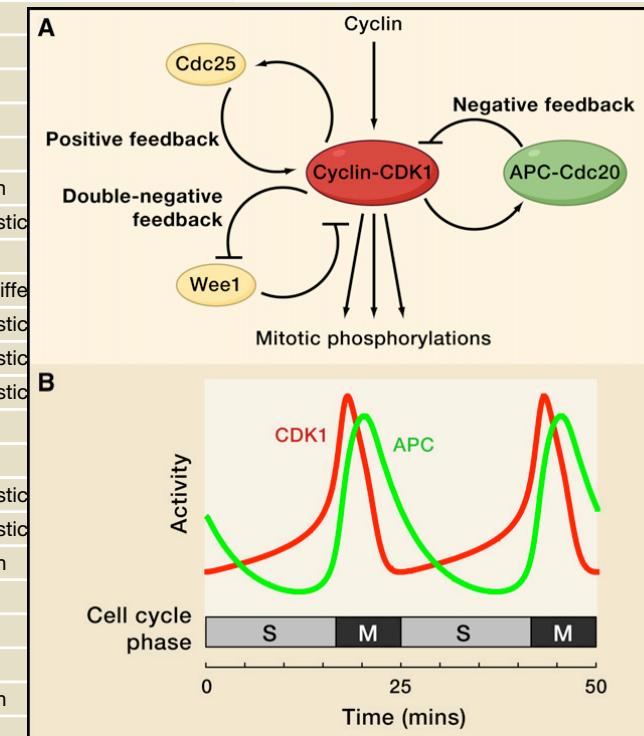
Terminalを起動



数理モデルを扱った論文

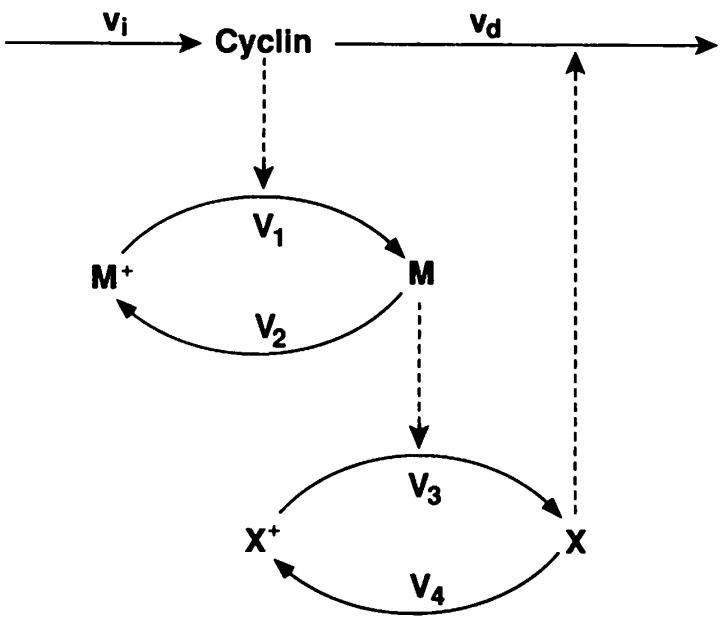


1970	No specific organism	ODE	(Sel'kov, 1970)
1974	No specific organism	ODE	(Gilbert, 1974)
1975	<i>Physarum polycephalum</i>	ODE	(Kauffman and Wille, 1975)
1975	<i>Physarum polycephalum</i>	ODE	(Tyson and Kauffman, 1975)
1991	<i>Xenopus laevis</i> embryos	ODE	(Goldbeter, 1991)
1991	<i>Xenopus</i> embryos	ODE	(Norel and Agur, 1991)
1991	<i>Xenopus</i> embryos, somatic cells	ODE	(Tyson, 1991)
1992	<i>Xenopus</i> embryos	ODE	(Obeyesekere et al., 1992)
1993	<i>Xenopus</i> embryos	ODE	(Novak and Tyson, 1993a)
1993	<i>Xenopus</i> embryos	ODE	(Novak and Tyson, 1993b)
1994	<i>Xenopus</i> embryos	ODE, delay differential equations	(Busenberg and Tang, 1994)
1996	<i>Xenopus</i> embryos	ODE	(Goldbeter and Guilmot, 1996)
1997	<i>S. pombe</i>	ODE	(Novak and Tyson, 1997)
1998	<i>S. pombe</i>	ODE	(Novak et al., 1998)
1998	<i>Xenopus</i> embryos	ODE	
1999	Mammalian somatic cells	ODE	
2003	<i>Xenopus</i> embryos	ODE	
2003	<i>S. cerevisiae</i>	ODE	
2004	<i>S. cerevisiae</i>	ODE	
2004	<i>S. cerevisiae</i>	Boolean	
2004	<i>S. pombe</i>	Stochastic	
2005	<i>Xenopus</i> embryos	ODE	
2006	Mammalian somatic cells	Delay differ.	
2006	<i>S. cerevisiae</i>	Stochastic	
2007	<i>S. cerevisiae</i>	Stochastic	
2007	<i>S. cerevisiae</i>	Stochastic	
2007	<i>S. cerevisiae</i>	Hybrid	
2008	<i>Xenopus</i> embryos	ODE	
2008	<i>S. cerevisiae</i>	Stochastic	
2008	<i>S. cerevisiae</i>	Stochastic	
2008	<i>S. pombe</i>	Boolean	
2008	Mammalian somatic cells	ODE	
2009	Mammalian somatic cells	ODE	
2010	<i>S. cerevisiae</i>	ODE	
2010	<i>S. cerevisiae, S. pombe</i>	Boolean	
2010	<i>S. pombe</i>	ODE	



Cell, 2011, 144(6): 874-85

数理モデルを扱った論文



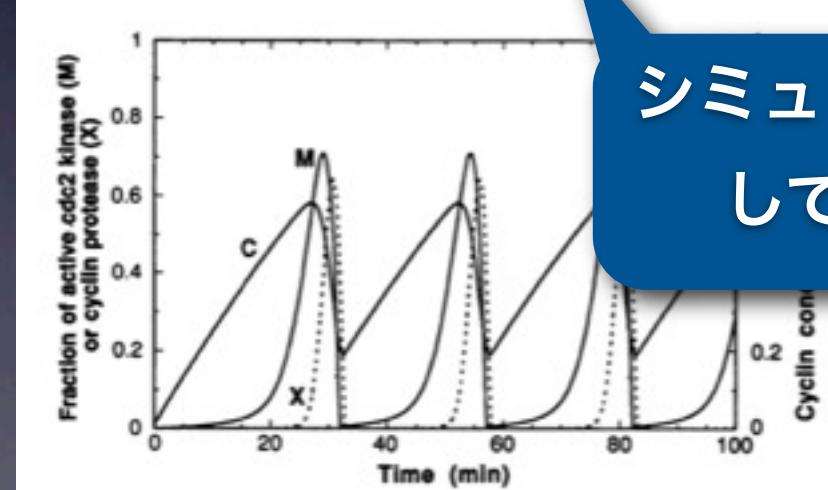
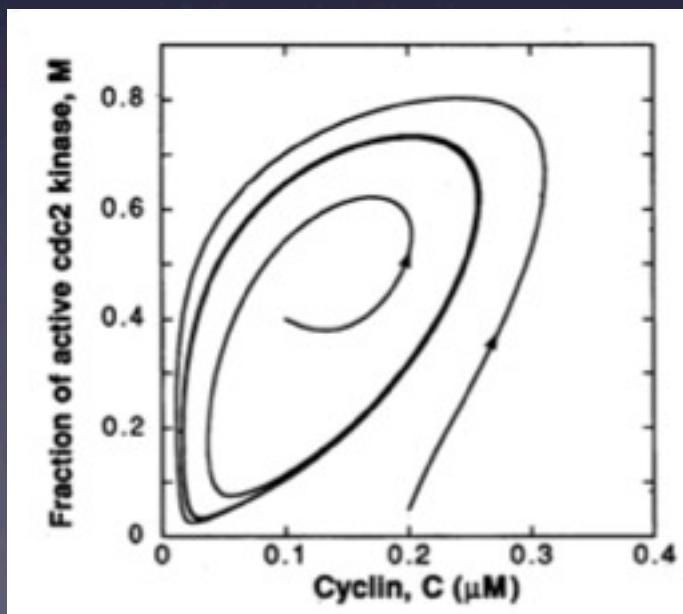
$$\frac{dC}{dt} = v_i - v_d X \frac{C}{K_d + C} - k_d C \quad [1]$$

$$\frac{dM}{dt} = V_1 \frac{(1 - M)}{K_1 + (1 - M)} - V_2 \frac{M}{K_2 + M}, \quad [1]$$

$$\frac{dX}{dt} = V_3 \frac{(1 - X)}{K_3 + (1 - X)} - V_4 \frac{X}{K_4 + X} \quad [1]$$

with

$$V_1 = \frac{C}{K_c + C} V_{M1}, \quad V_3 = M V_{M3}. \quad [2]$$

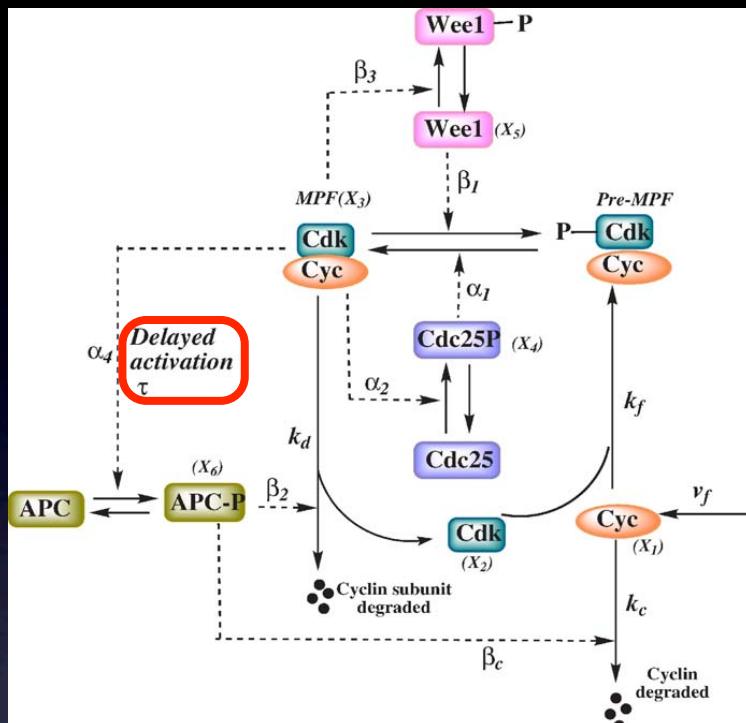


シミュレーション
してみたい!

PNAS, 1991, 88(20): 9107-9111.

数理モデルを扱った論文

シミュレーション
してみたい!



$$\frac{dX_1}{dt^a} = v_f - k_f X_1 X_2 - (k_c + \beta_c X_6) X_1$$

$$\frac{dX_2}{dt} = k_d [1 + \beta_2 X_6] X_3 - k_f X_1 X_2$$

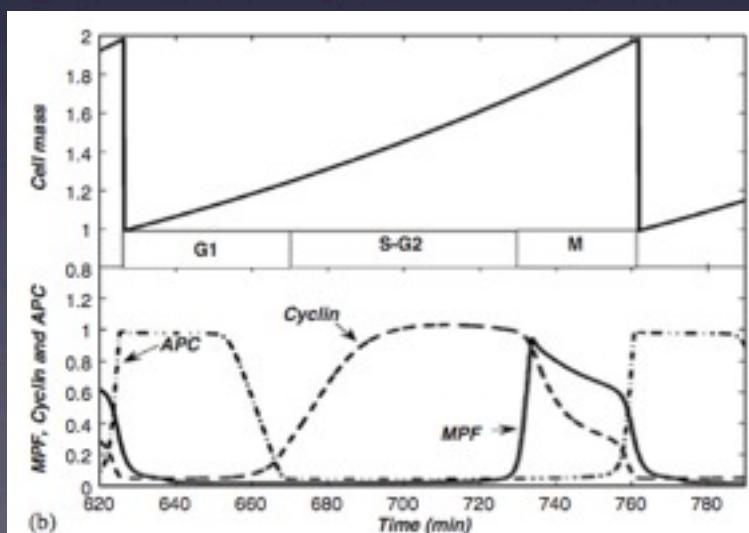
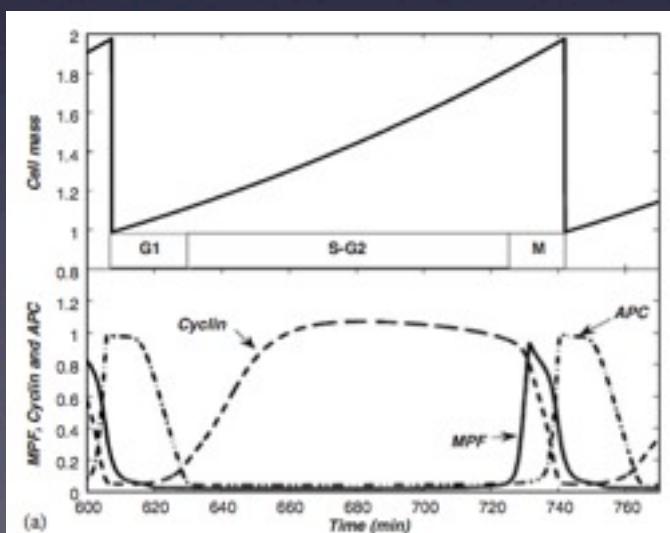
$$\frac{dX_3}{dt} = v_{M1} \left[1 + \alpha_1 \frac{X_4}{K_a + X_4} \right] \frac{(C - X_2 - X_3)}{J_1 + (C - X_2 - X_3)} - v'_{M1} \left[1 + \alpha_4 \frac{m X_3(t-\tau)}{K_a + m X_3(t-\tau)} \right] \frac{X_1}{J'_1 + X_1} - \alpha_2 X_3 X_4$$

$$\frac{dX_4}{dt} = v_{M2} \left[1 + \alpha_2 \frac{m X_3}{K_a + m X_3} \right] \frac{(1 - X_4)}{J_2 + (1 - X_4)} - v'_{M2} \frac{X_4}{J'_2 + X_4}$$

$$\frac{dX_5}{dt} = v_{M3} \frac{(1 - X_5)}{J_3 + (1 - X_5)} - v'_{M3} \left[1 + \beta_3 \frac{m X_3}{K_a + m X_3} \right] \frac{X_5}{J'_3 + X_5}$$

$$\frac{dX_6}{dt} = v_{M4} \left[1 + \alpha_4 \frac{m X_3(t-\tau)}{K_a + m X_3(t-\tau)} \right] \frac{(1 - X_6)}{J_4 + (1 - X_6)} - v'_{M4} \frac{X_6}{J'_4 + X_6}$$

$$\frac{dm}{dt} = \mu' m (1 - m/a)$$



J Theor Biol, 2006, 241(3): 617-27

シミュレータの現状

	Recent contact	Creation	Simulation	Analysis	Database	Utility	ODE	DAE	PDE	Stochastic	Events	Logical	Other	Frameworks	API	Dep.	Platforms	SBML	Available.	
																	Linux	Mac OS X	Windows	
Cain	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	
CARMEN	•	•	•	•													•	•	•	•
Cell Illustrator	•	•	•														•	•	•	•
CellDesigner	•	•	•				•	•									•	•	•	•
Cellerator	•	•	•				•	•									•	•	•	•
CellMC																	•	•	•	•
CellML2SBML																	•	•	•	•
CellNetAnalyzer	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
Cellware																	•	•	•	•
CLEML																	•	•	•	•
CL-SBML	•																•	•	•	•
COBRA	•																•	•	•	•
CompuCell3D	•						•	•									•	•	•	•
ConsensusPathDB																	•	•	•	•
COPASI	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•

どれを選べばいい?

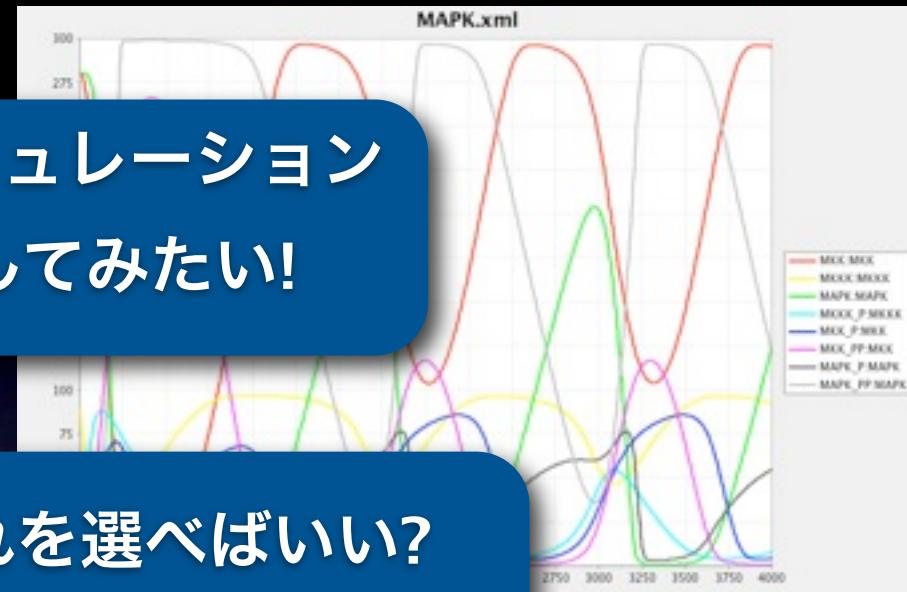


<http://sbml.org/>

今日の目標

- 数理モデルの構築
- シミュレータの実装
- シミュレーション

シミュレーション
してみたい!



どれを選べばいい?

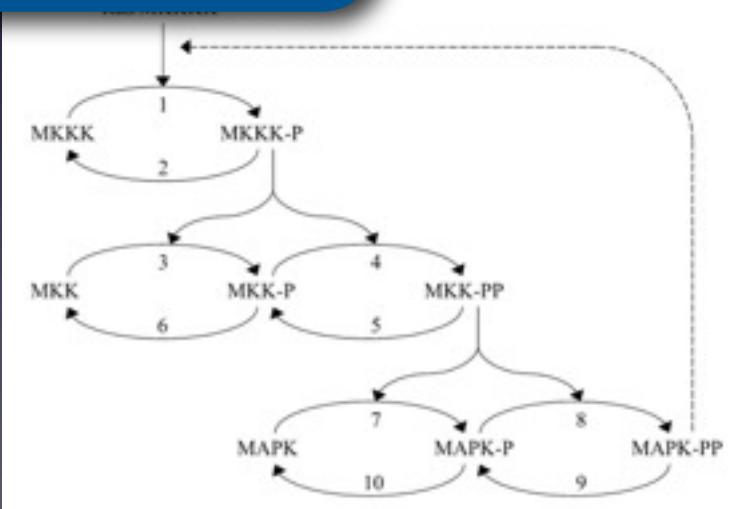
Table 1. Kinetic equations comprising the computation

Reaction number	Rate equation
1	$V_1 \cdot [MKKK] / ((1 + ([MAPK-PP]/K_1)^n) \cdot (K_1 + [MKKK]))$
2	$V_2 \cdot [MKKK-P] / (K_2 + [MKKK-P])$
3	$k_3 \cdot [MKKK-P] \cdot [MKK] / (K_3 + [MKK])$
4	$k_4 \cdot [MKKK-P] \cdot [MKK-P] / (K_4 + [MKK-P])$
5	$V_5 \cdot [MKK-PP] / (K_5 + [MKK-PP])$
6	$V_6 \cdot [MKK-P] / (K_6 + [MKK-P])$
7	$k_7 \cdot [MKK-PP] \cdot [MAPK] / (K_7 + [MAPK])$
8	$k_8 \cdot [MKK-PP] \cdot [MAPK-P] / (K_8 + [MAPK-P])$
9	$V_9 \cdot [MAPK-PP] / (K_9 + [MAPK-PP])$
10	$V_{10} \cdot [MAPK-P] / (K_{10} + [MAPK-P])$

Total concentrations: $[MKKK]_{total} = 100$; $[MKK]_{total} = 300$; $[MAPK]_{total} = 300$

$$[MKK]_{total} = [MKK] + [MKK-P] + [MKK-PP]$$

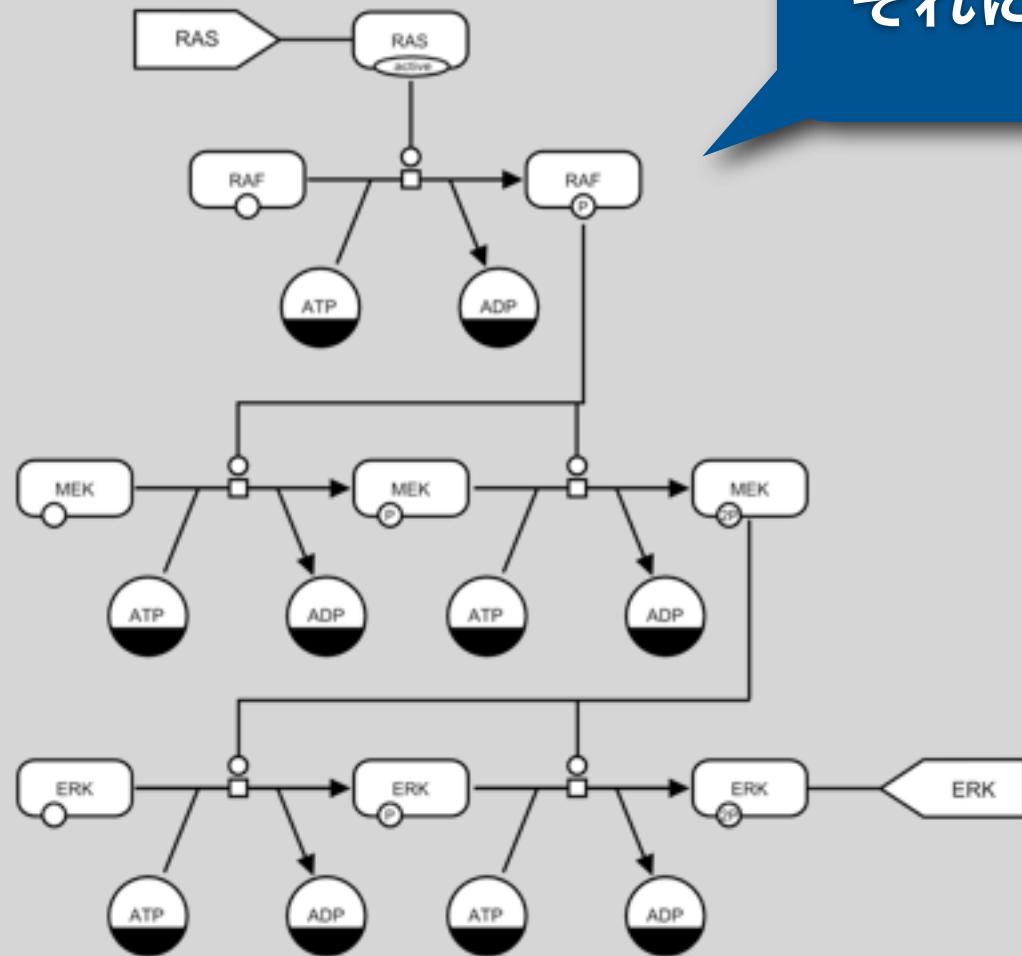
$$[MAPK]_{total} = [MAPK] + [MAPK-P] + [MAPK-PP]$$



モデル

対象としている現象

それに関わる分子・生化学反応



$$\frac{dx}{dt} = kx$$

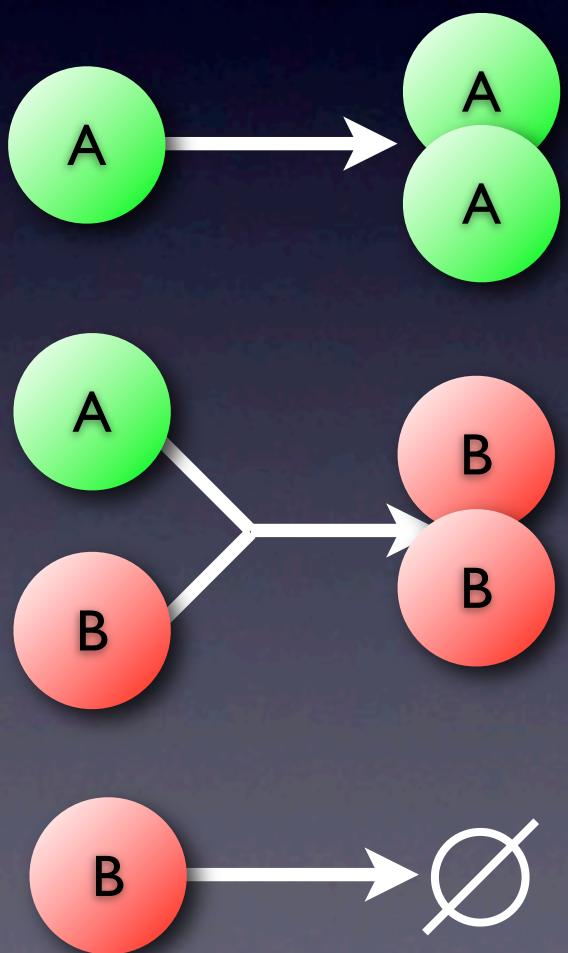


モデル

- シミュレーションしたい対象(現象)
- 抽象化
- 未来の予測

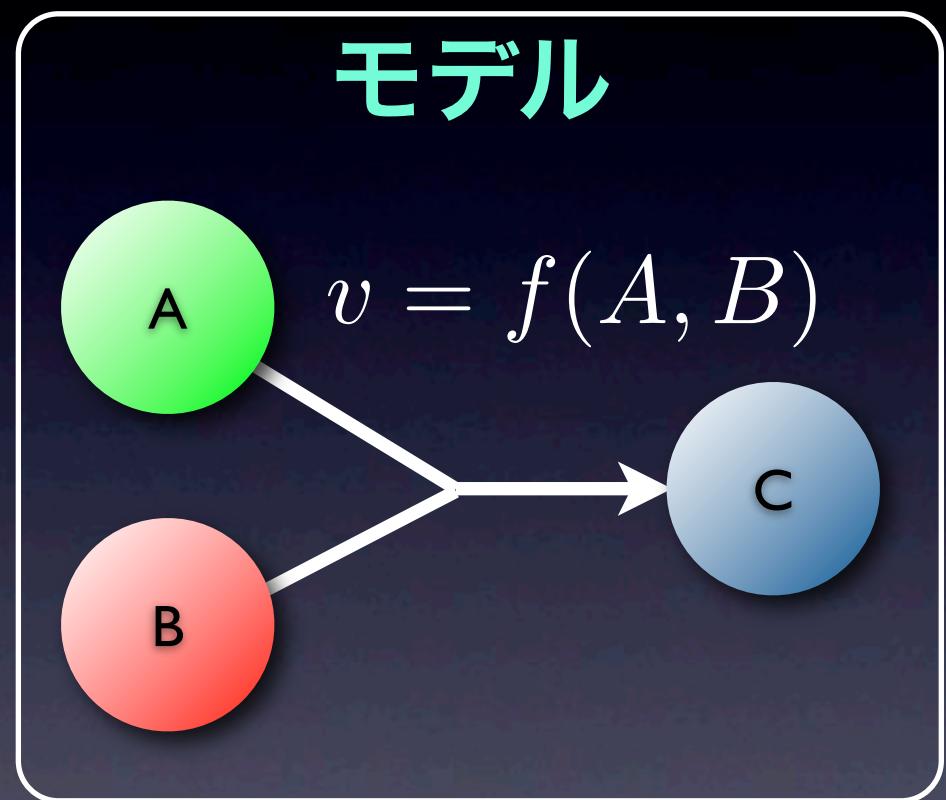


Lotka-Volterra model



モデル

- 反応方程式
- 分子濃度・数の増減
- 分子濃度(初期値)

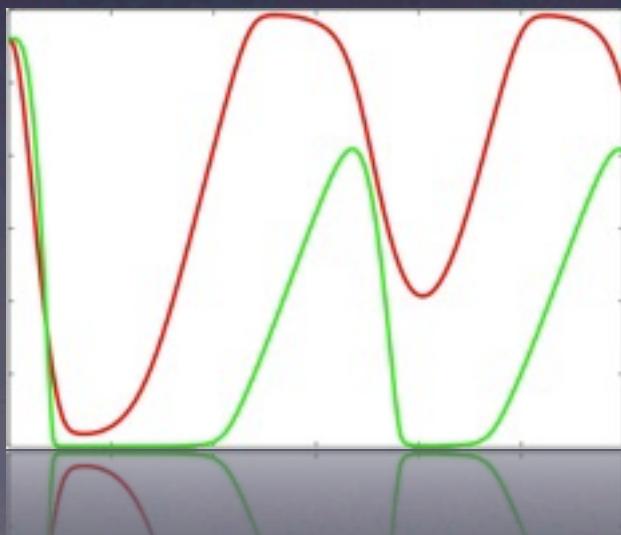


モデル内のすべての分子濃度・反応方程式が定義されていればシミュレーション可能

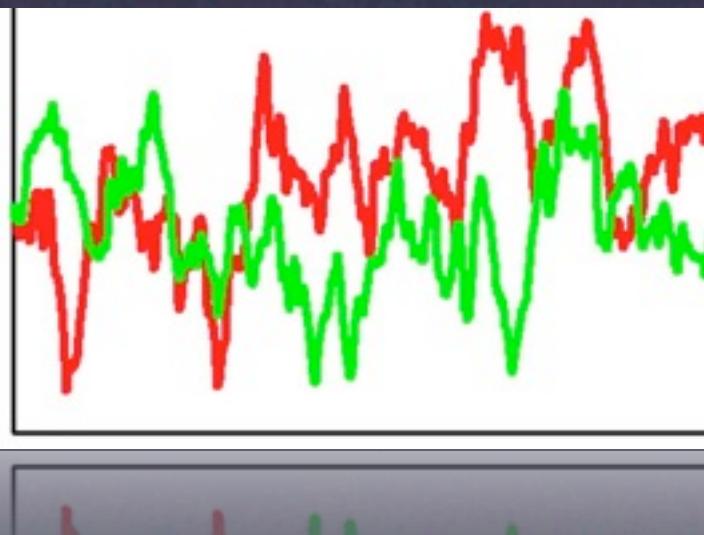
シミュレーション

- 分子濃度を記述した常微分方程式 (ODE)
- 分子数の確率的な変化を記述した確率モデル (SSA)
- 分子濃度の空間的分布を記述した偏微分方程式 (PDE)

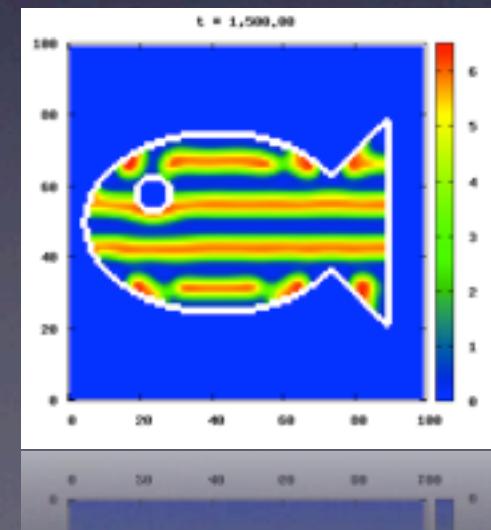
ODE



SSA

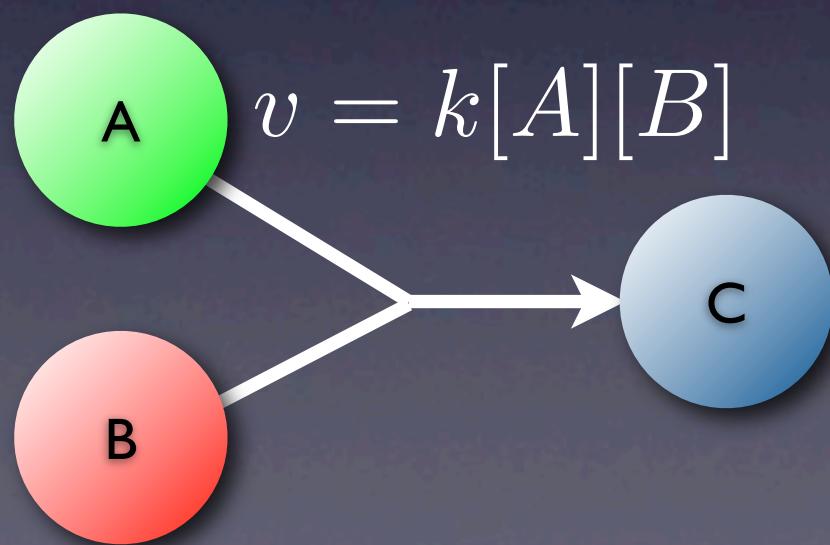


PDE



常微分方程式(ODE)

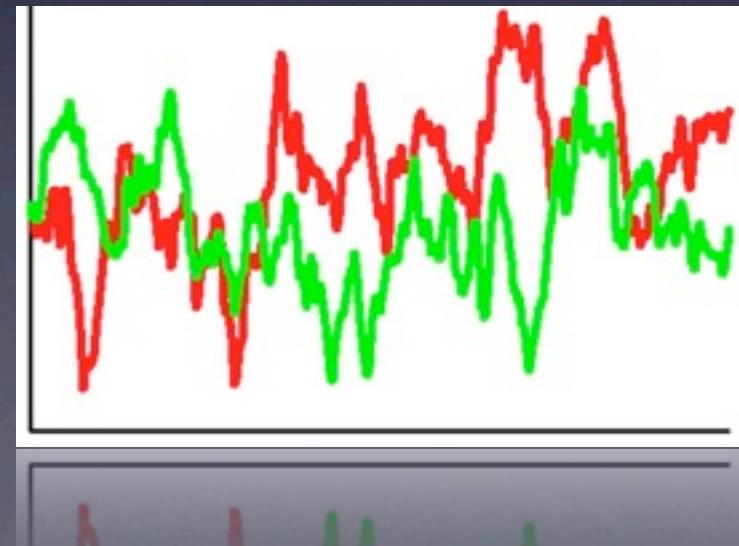
- 分子濃度の変化をモデル化
- 各反応による分子濃度の変化を記述
- 連続的な値を取り扱う
- 決定論的
- 数値積分



確率モデル(SSA)

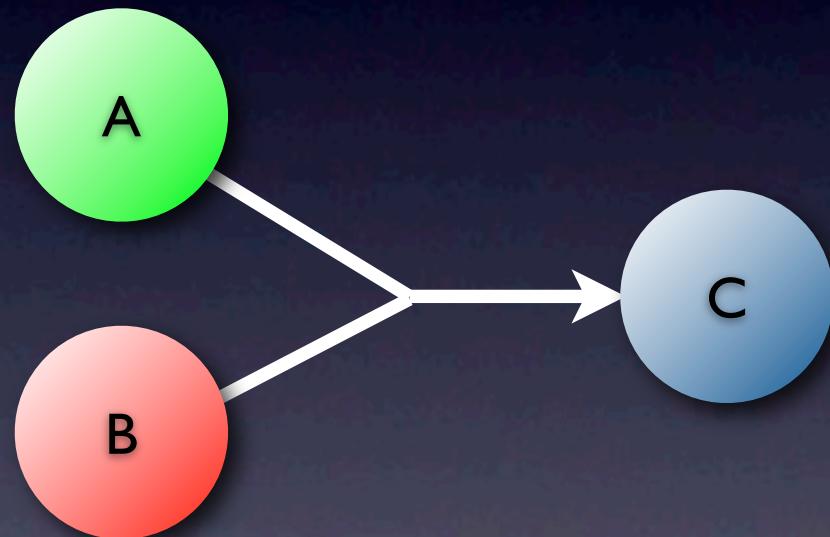
- 分子数の変化をモデル化
- 各反応による分子数の変化を記述
- 連続的な値ではなく、離散値を取り扱う
- 非決定論的

SSA

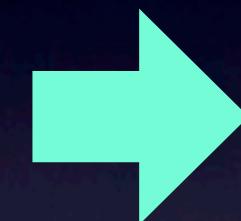


確率モデル(SSA)

- 分子数の変化をモデル化

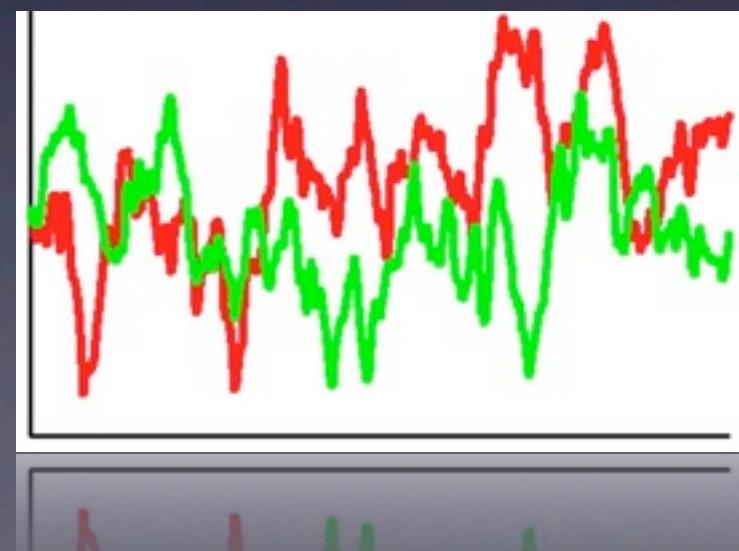


A: 300
B: 200
C: 0



A: 298
B: 198
C: 2

SSA



偏微分方程式(PDE)

分子濃度: 高

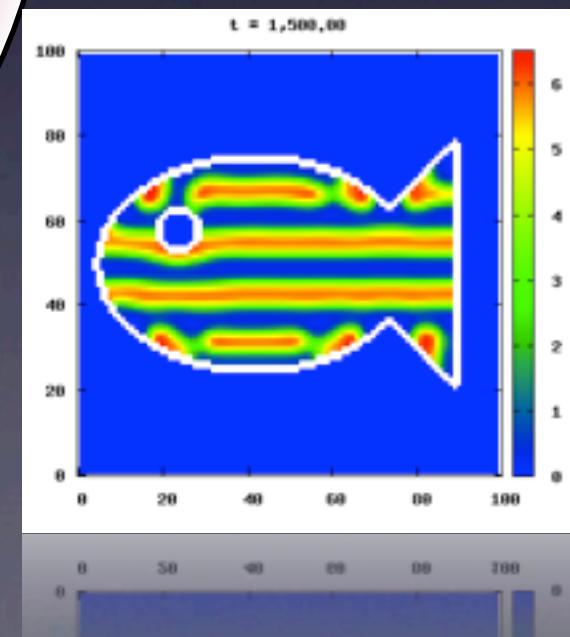
分子の拡散

分子の移流

境界での
膜輸送

細胞質

分子濃度: 低



ODE? SSA? PDE?



REVIEW ARTICLE

Applications and trends in systems biology in biochemistry

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Keywords

metabolism; modeling; quantitative experiments; signaling; simulation; systems biology

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doi:10.1111/j.1742-4658.2011.08217.x

Introduction

One of the fastest growing fields in the life sciences is systems biology. PubMed lists more than 3000 articles which, in one way or the other, use this term in their title or abstract during the last decade (precisely, the last 11 years, including the year 2000) compared to a mere three articles in the preceding century. Obviously, this is partially a result of the fact that the term 'systems biology' had not been used during that time. However, as we will see in the present review, also with respect to research that would now be called systems biology, there is clearly significantly less to report before the year 2000. Interestingly, looking closely at the more than 3000 articles using the term 'systems biology', it becomes apparent that approximately half of them describe methodological work either on the computational or the experimental side, and more than one-third are classified as reviews. However, only a

handful of the latter represent reviews that actually review a set of articles. Most of the articles classified as reviews could rather be classified as news and views. Another large portion of articles uses the term 'systems biology' in a different sense than we would understand it (e.g. stating that they are investigating a biological system and it is therefore systems biology). This latter point necessitates the definition of the term 'systems biology' as we (the authors) understand it, as outlined below.

Systems biology combines quantitative experimental data from complex molecular networks (e.g. biochemistry, cell biology in the living cell) with computational modeling. Here, computational modeling does not refer to statistical models or models of data mining but rather to a mathematical or 'virtual' representation of the living system of interest in the computer, where

Abbreviations

FBA, flux balance analysis; ODE, ordinary differential equation; PDE, partial differential equation.

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2767

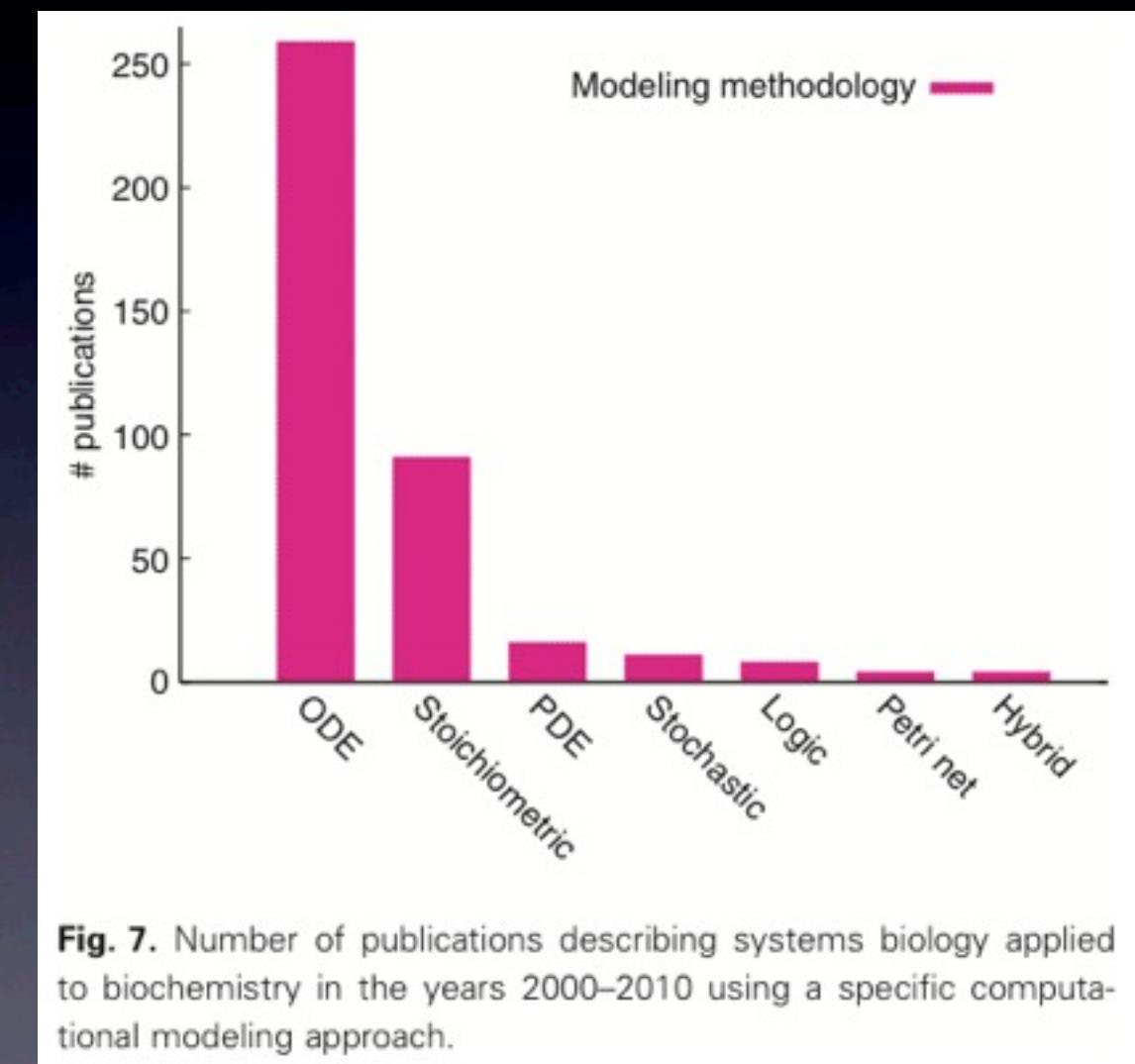


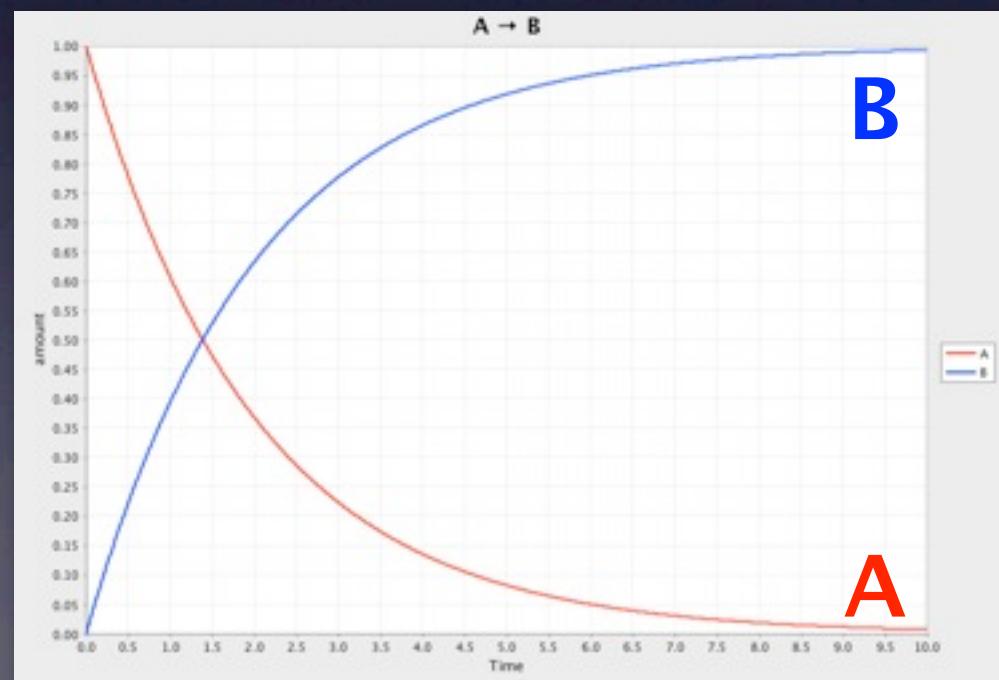
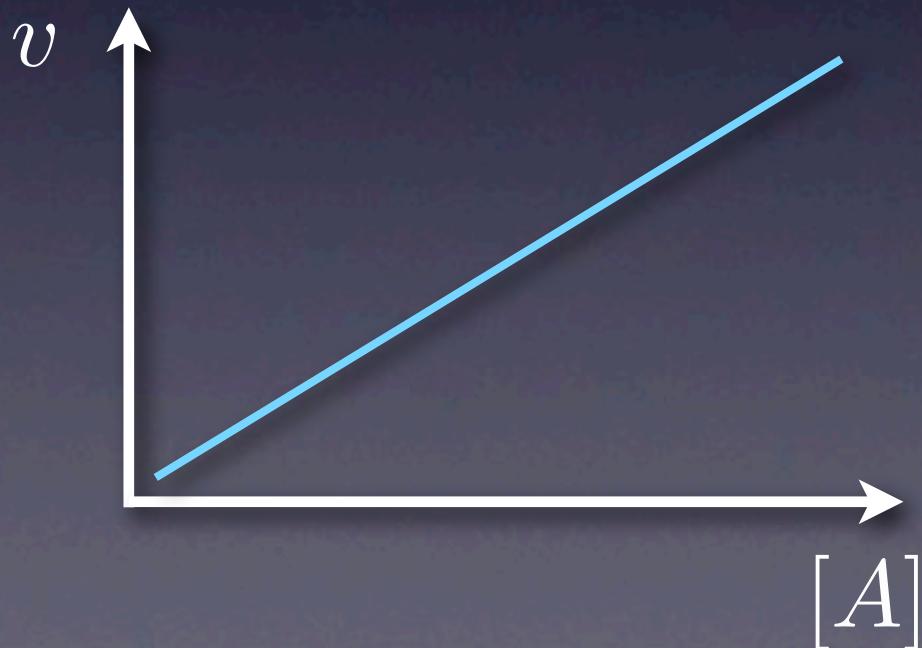
Fig. 7. Number of publications describing systems biology applied to biochemistry in the years 2000–2010 using a specific computational modeling approach.

ODEで書かれた反応方程式

$$v = k[A]$$



$$\frac{d[A]}{dt} = -k[A]$$
$$\frac{d[B]}{dt} = k[A]$$



Mass-action

$$v_1 = k_1[A] \quad v_2 = k_2[B]$$



$$\frac{d[B]}{dt} = k_1[A] - k_2[B]$$

大事なこと

$$v_1 = k_1[A] \quad v_2 = k_2[B]$$



$$\frac{d[B]}{dt} = k_1[A] - k_2[B]$$

- 和、差で反応毎に分解できる

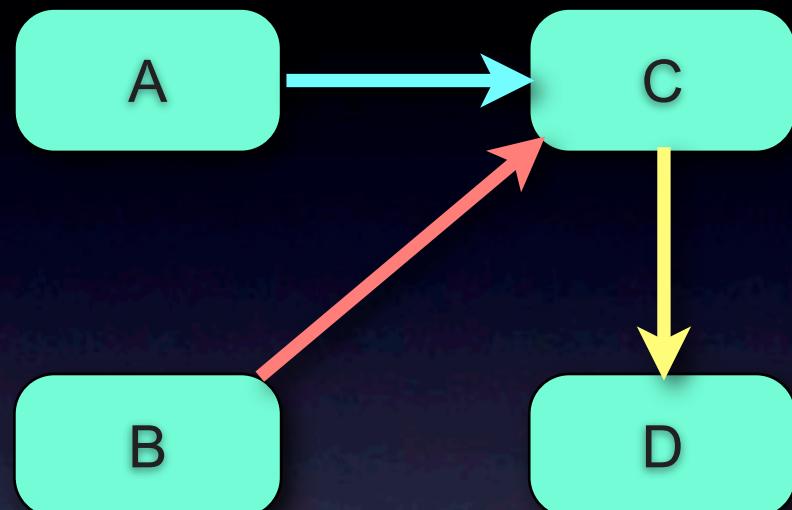
ODE ↔ ネットワーク

$$\frac{dA}{dt} = -k_1 A$$

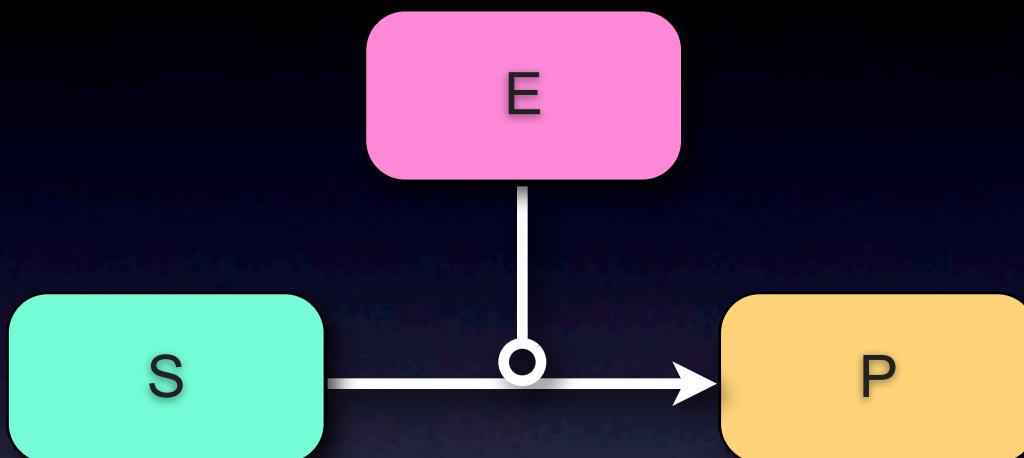
$$\frac{dB}{dt} = -k_2 B$$

$$\frac{dC}{dt} = k_1 A + k_2 B - k_3 C$$

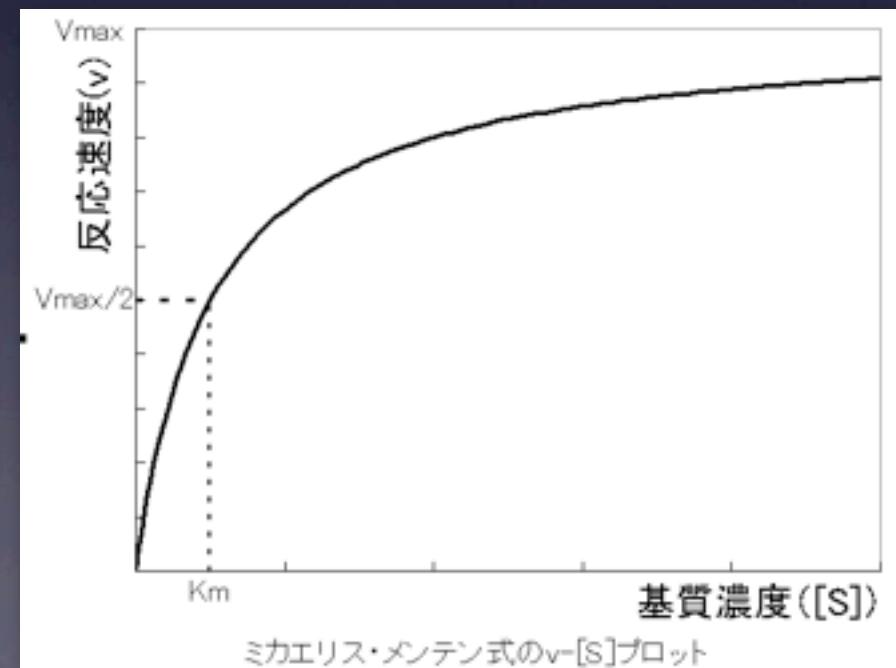
$$\frac{dD}{dt} = k_3 C$$



Michaelis–Menten



$$\frac{d[P]}{dt} = \frac{V_{max}[S]}{K_m + [S]}$$



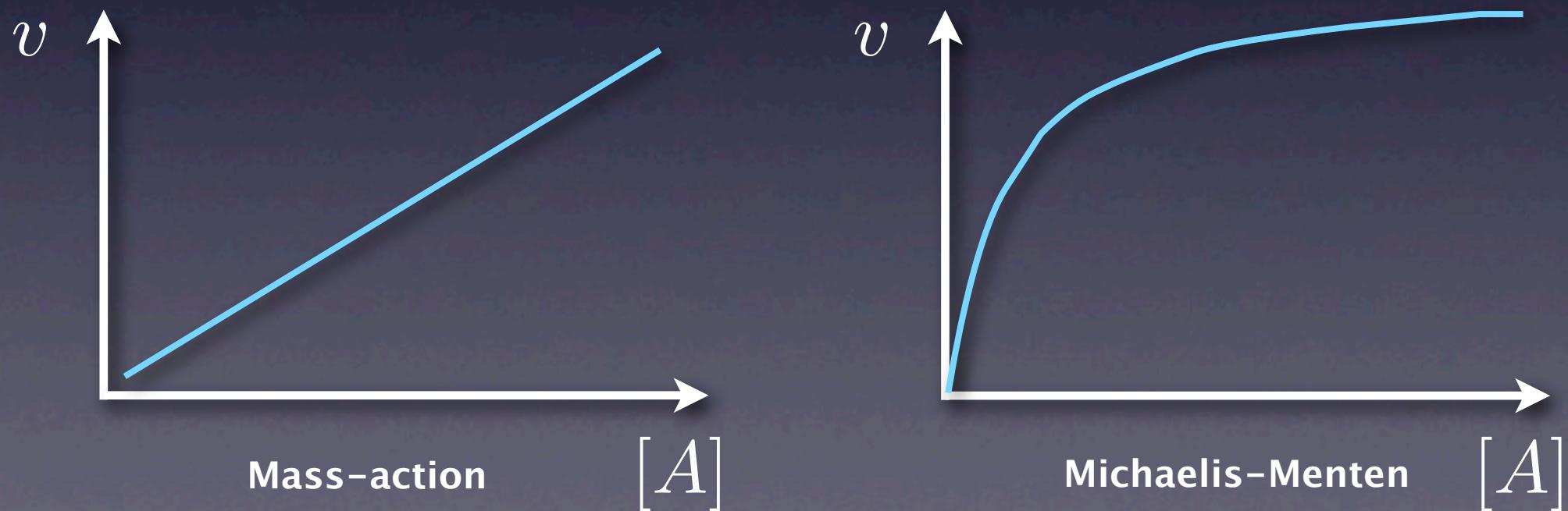
Mass-action

$$v = k[A]$$

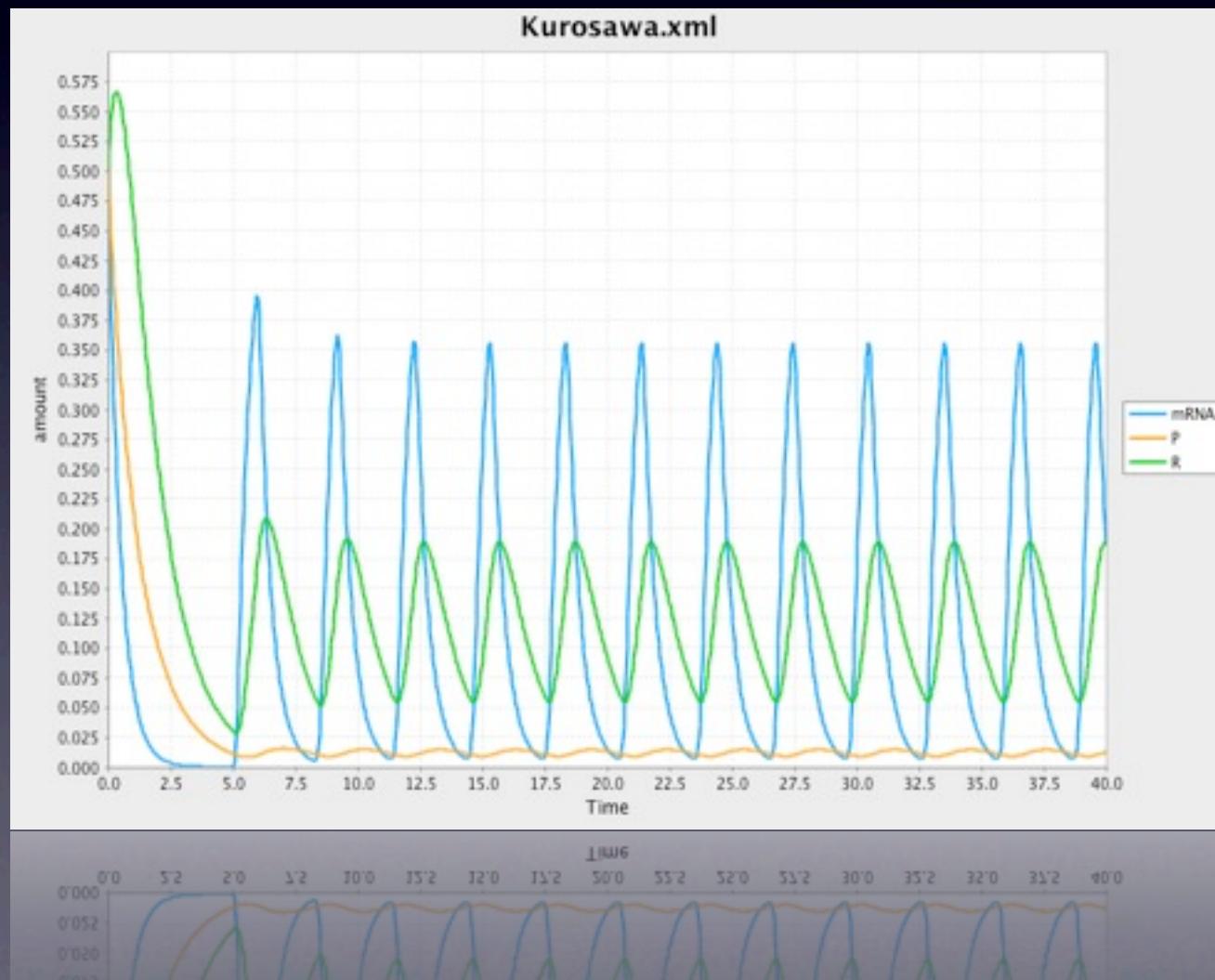


$$\frac{d[A]}{dt} = -k[A]$$

$$\frac{d[B]}{dt} = k[A]$$

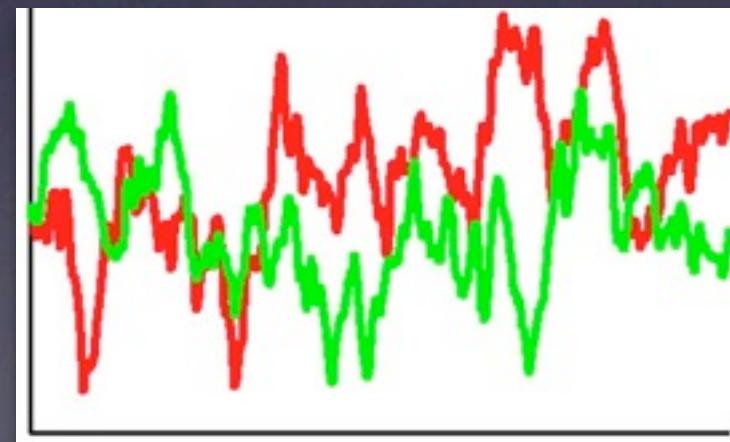
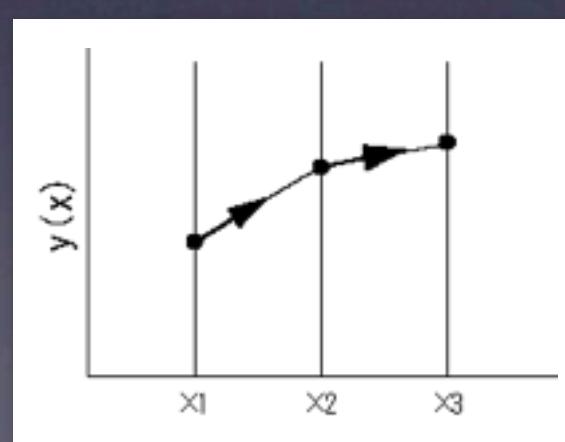


ODEシミュレータの実装



Why Simulate a model?

- 実験で(検証する前に/検証できない事項を)予測したい
- 未来 (ある時間: t) の状態を予測



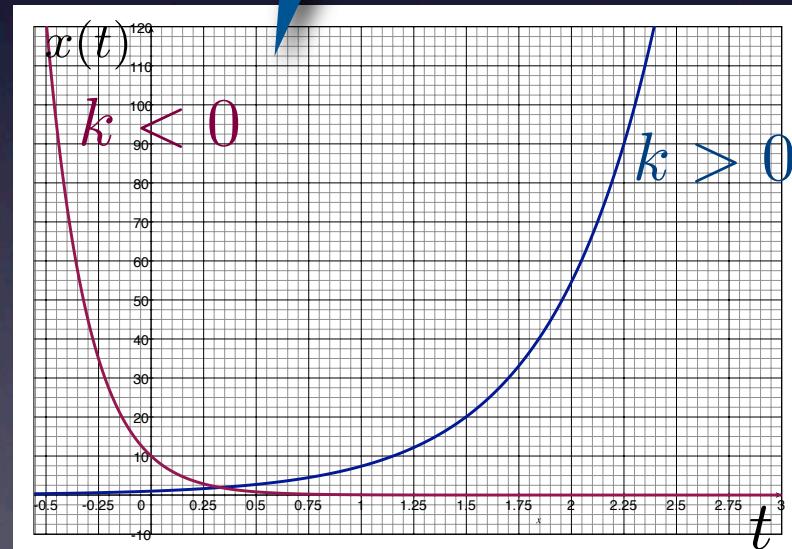
未来の予測

- 未来 (ある時間: t) の状態を予測



解析的に未来を予測

$$\frac{dx}{dt} = kx$$



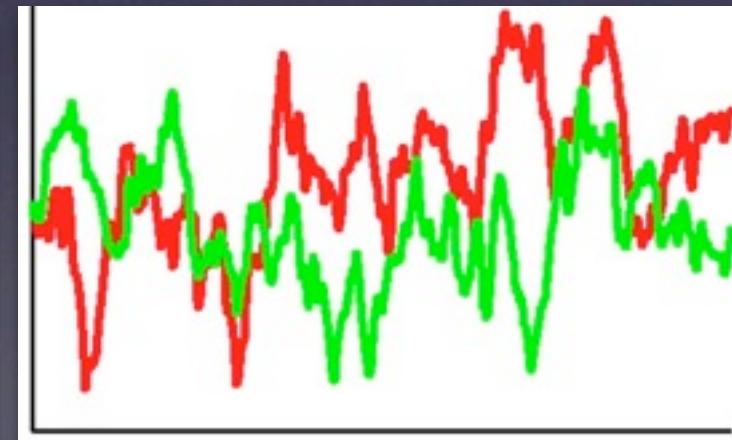
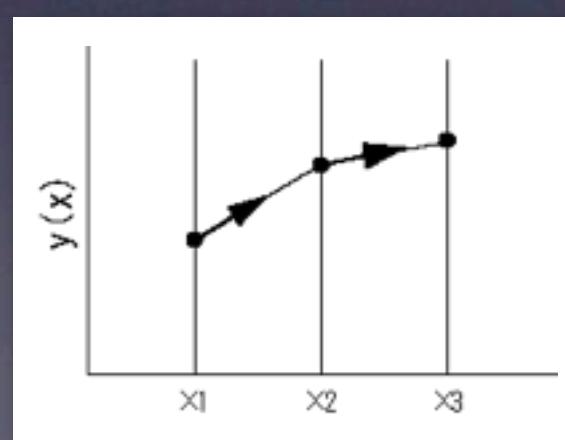
$$x = C_1 e^{kt}$$

数値計算が必要な場合

- 解析的に解けない時
- x_{t+1} を x_t から求めるしかない時



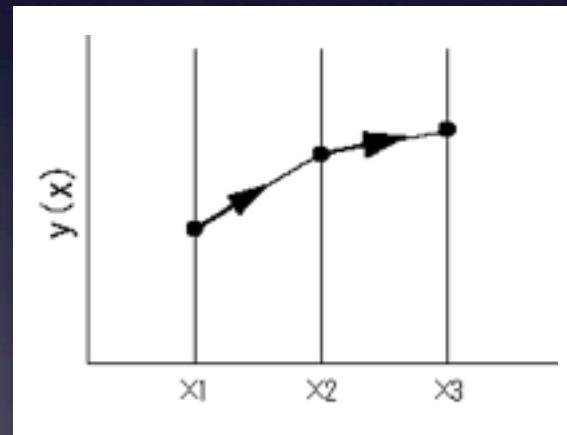
- 数値計算
- 確率モデル



数値計算が必要な場合

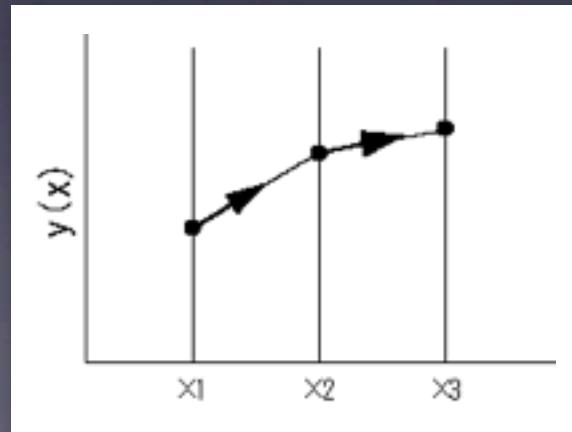
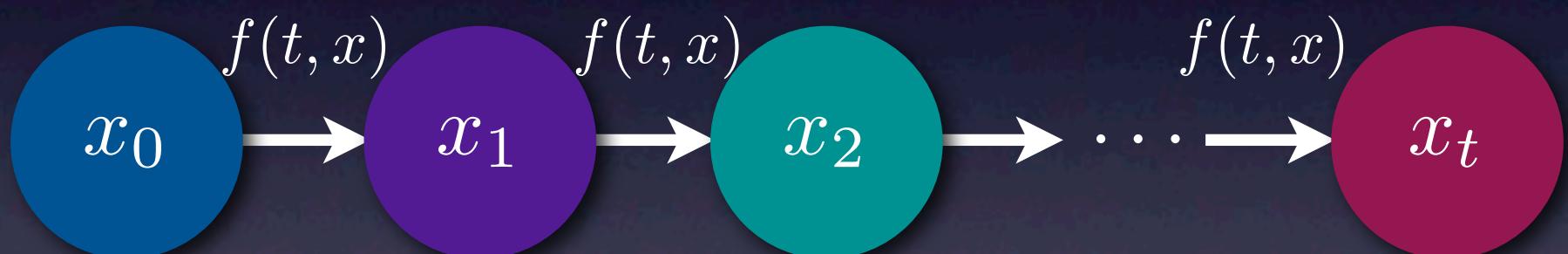
- 多くの場合、連立微分方程式は解析的に解けない
→ 積分できない
- 数値積分が必要

ODEシミュレータは
数値積分を行なっている



ODEの数値積分

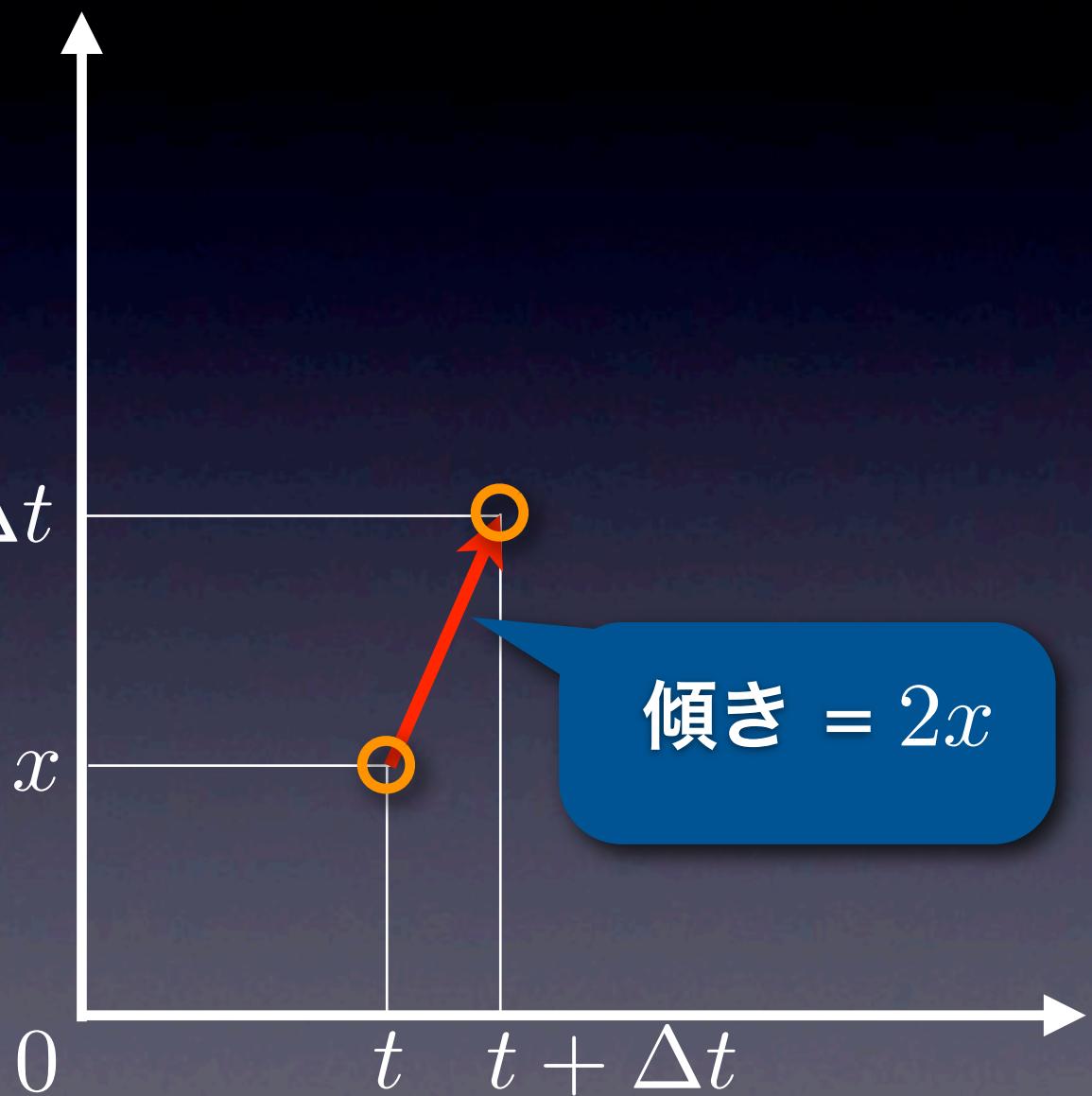
$$\frac{dx}{dt} = f(t, x)$$



ODE Simulatorの作り方

$$\frac{dx}{dt} = 2x$$

$$x + 2x\Delta t$$



ODE Simulatorの作り方

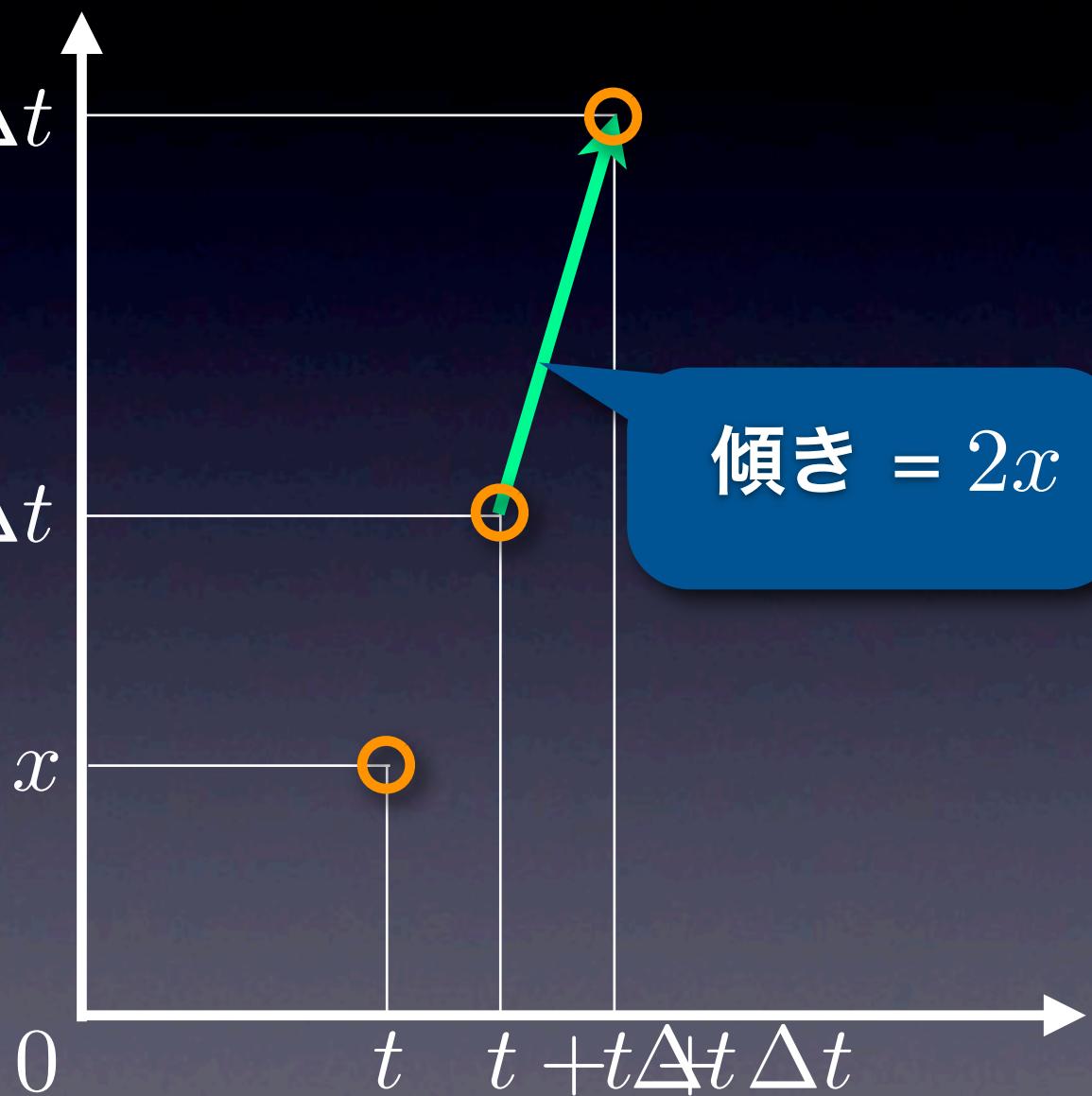
$$\frac{dx}{dt} = 2x$$

$$x + 2x\Delta t$$

$$x + 2x\Delta t$$

オイラー法

Euler's Method



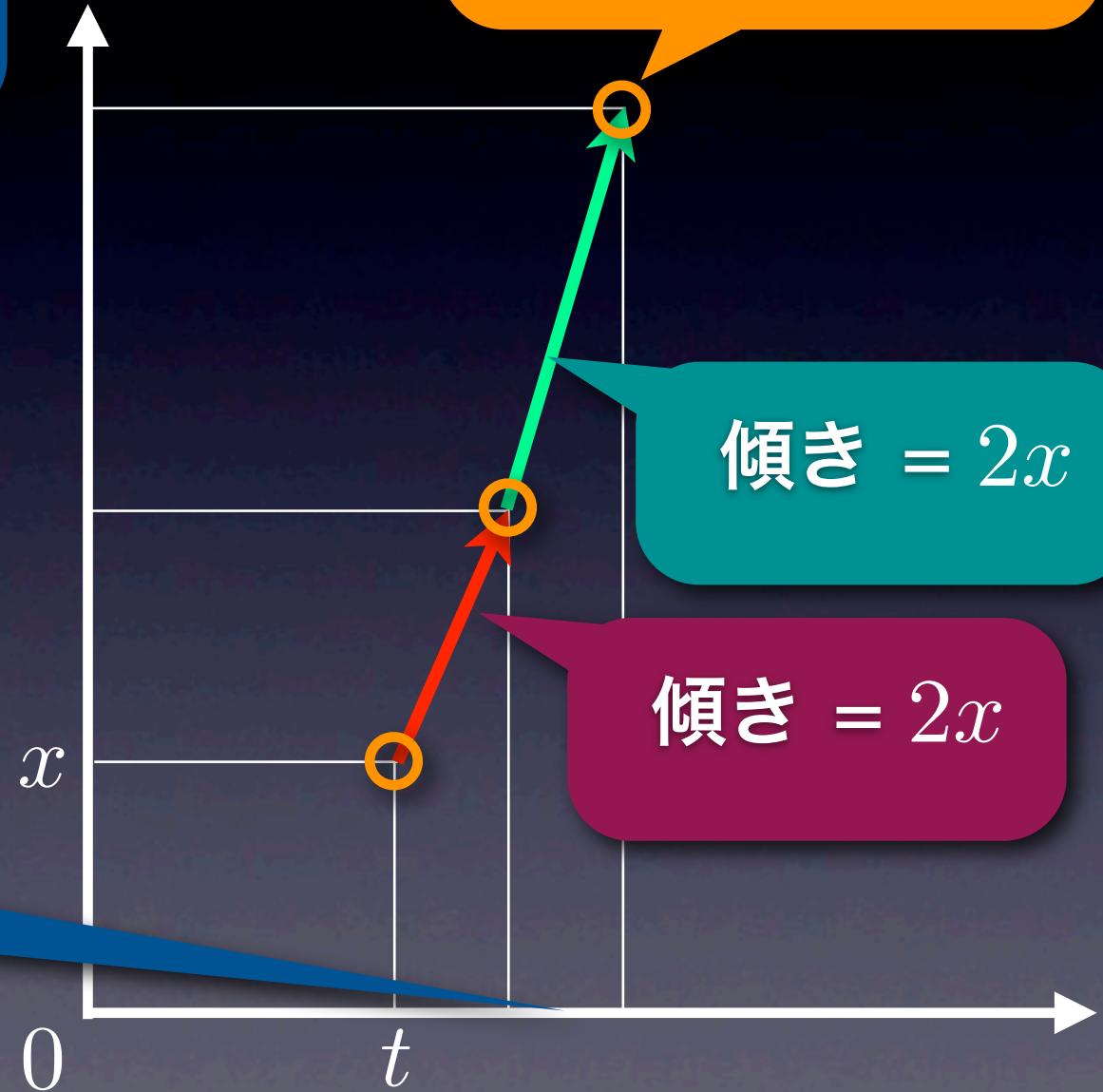
ポイント

x は dx に応じて変化

$$\frac{dx}{dt} = 2x$$

t は Δt で増加

以降、繰り返し



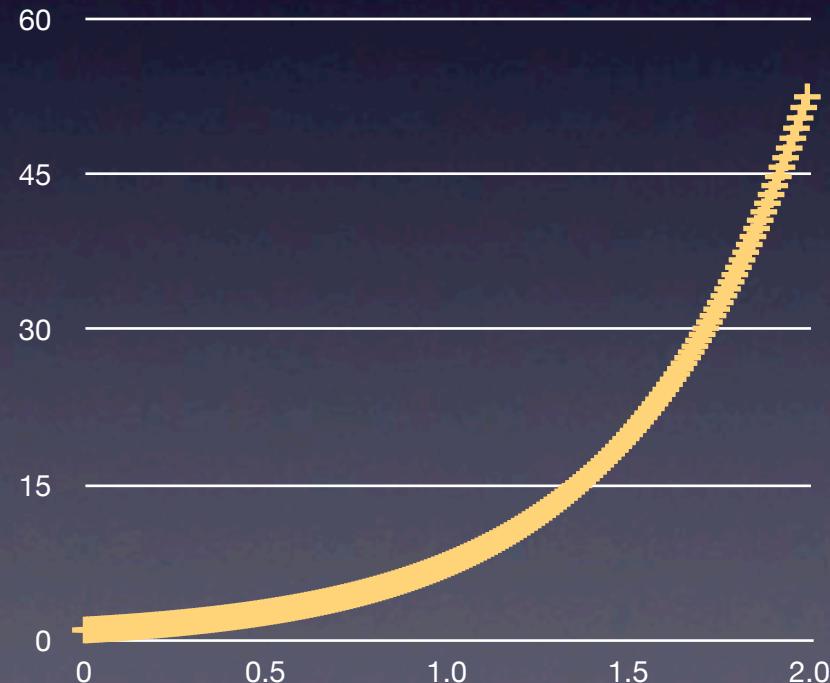
ODE Simulator

- $\frac{dx}{dt} = 2x$ を解くシミュレータ $t = 0, x = 1.0$

```
#!/usr/bin/perl
```

```
$dt = 0.01;  
$t = 0.0;  
$x = 1.0;  
  
for ($i = 0; $i <= 200; $i++) {  
    print "$t,$x\n";  
    $dx = 2 * $x * $dt;  
    $x = $x + $dx;  
    $t = $t + $dt;  
}
```

シミュレーション結果



PerlでCODEシミュレータ

Windowsの人

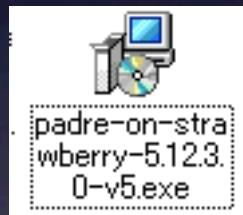
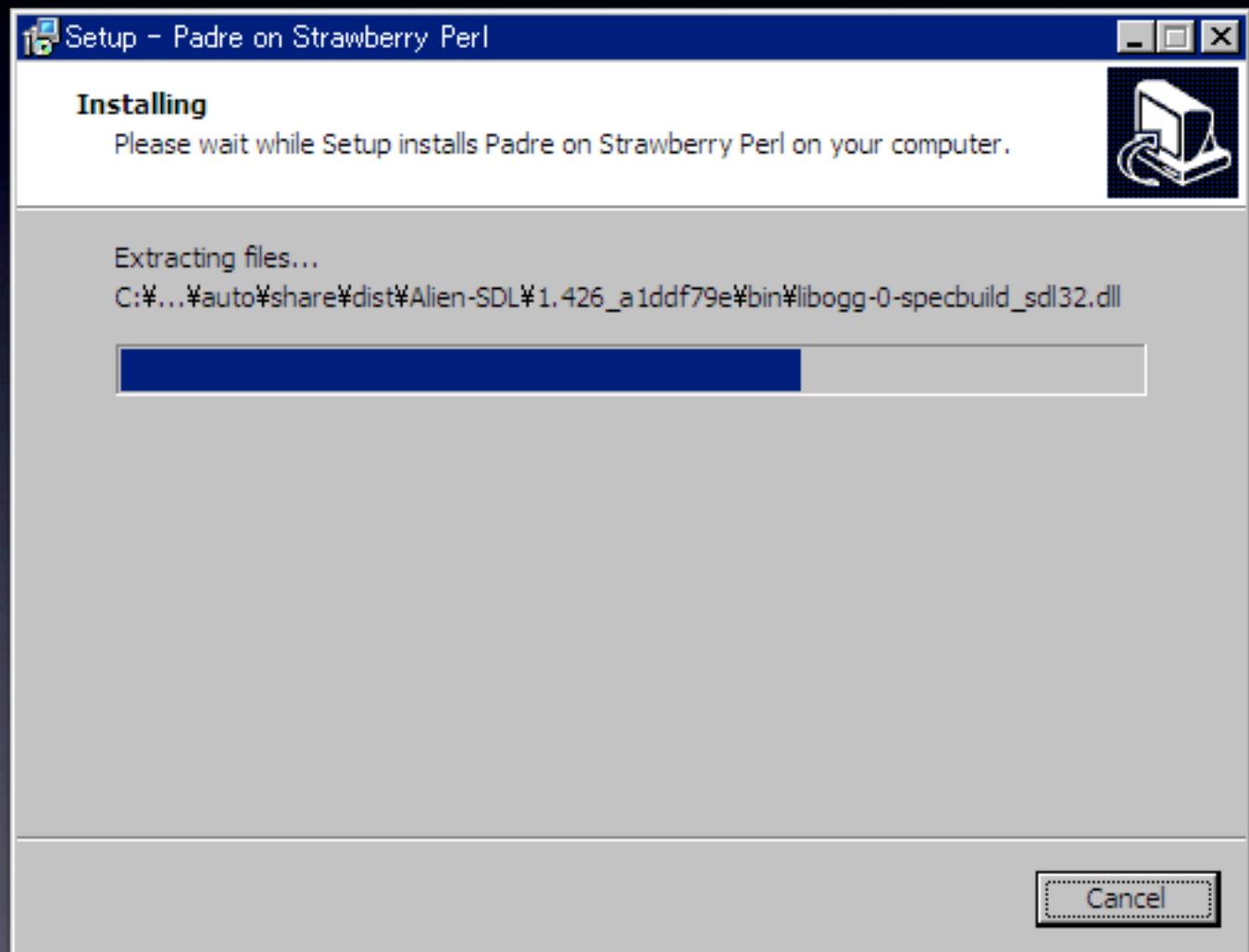
<http://padre.perlide.org>

The screenshot shows a web browser window displaying the official website for Padre, the Perl IDE. The title bar reads "Padre, the Perl IDE". The address bar shows the URL "padre.perlide.org". The main content area features a large blue butterfly logo on the right. The text on the left describes Padre as a Perl IDE, suitable for both new Perl programmers and advanced developers. It highlights its extensible plugin system and support for multi-lingual and multi-technology projects. A "Download" button with a blue arrow icon is prominently displayed, and it is circled with a red line. Below the download button, a section titled "Features" lists several key capabilities: customizable syntax highlighting, syntax checking for Perl 5 and Perl 6, refactoring tools, context-sensitive help, code completion, and being beginner-friendly.

- Beginner-friendly
- Context-sensitive help and code completion
- Syntax checking for Perl 5 and Perl 6
- Refactoring tools for Perl 5 and Perl 6
- Customizable syntax highlighting for many languages and visual editor effects

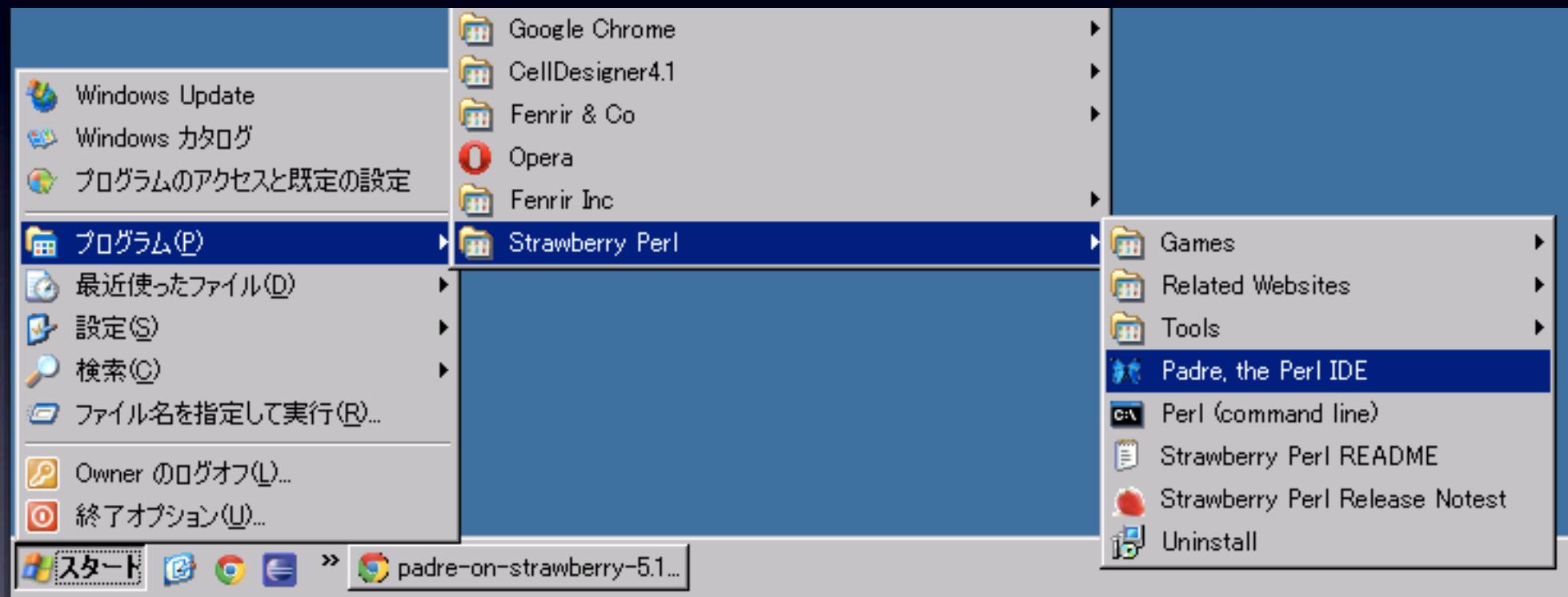
Padreをinstall

<http://padre.perlide.org>

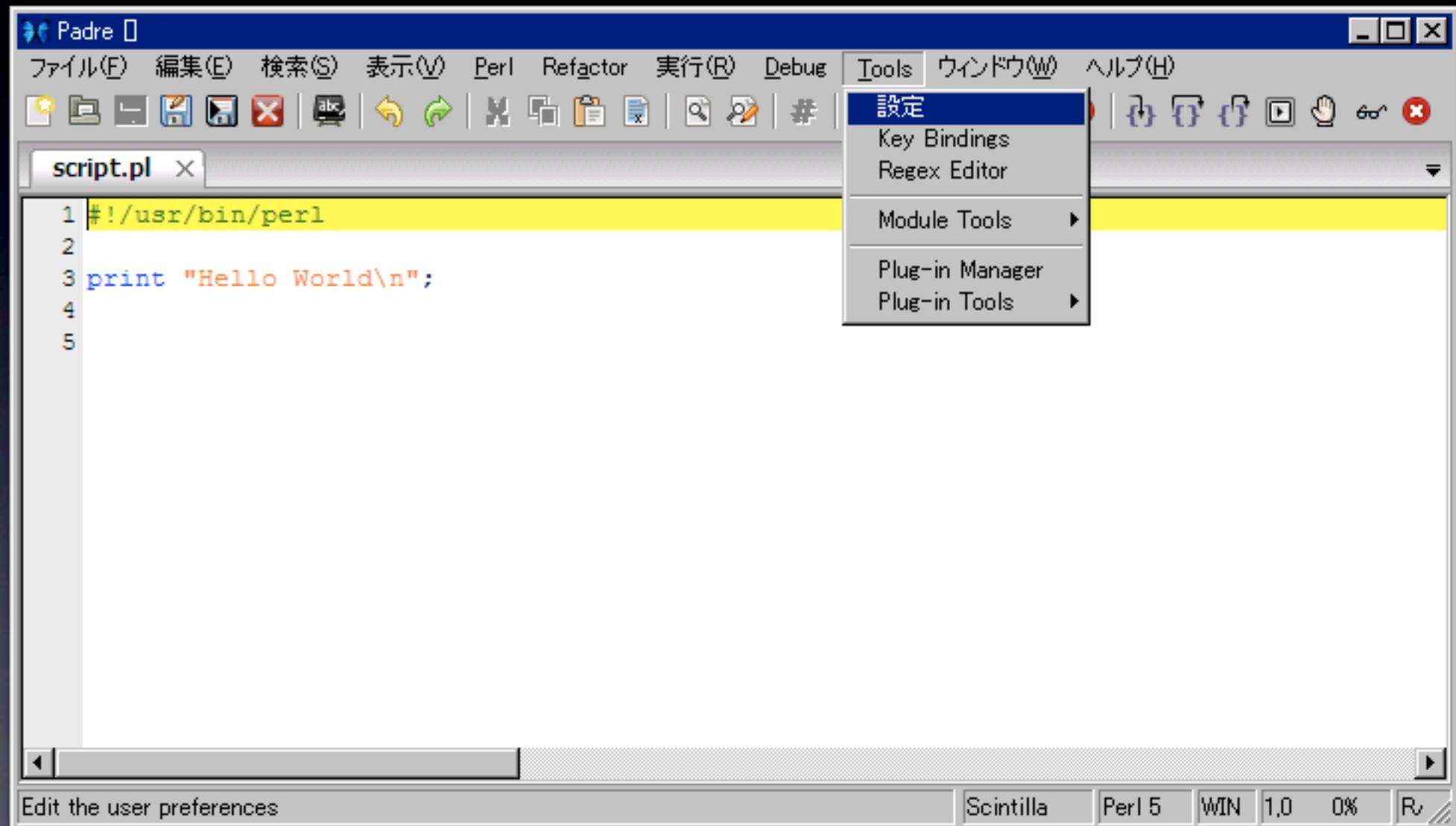


Padreを実行

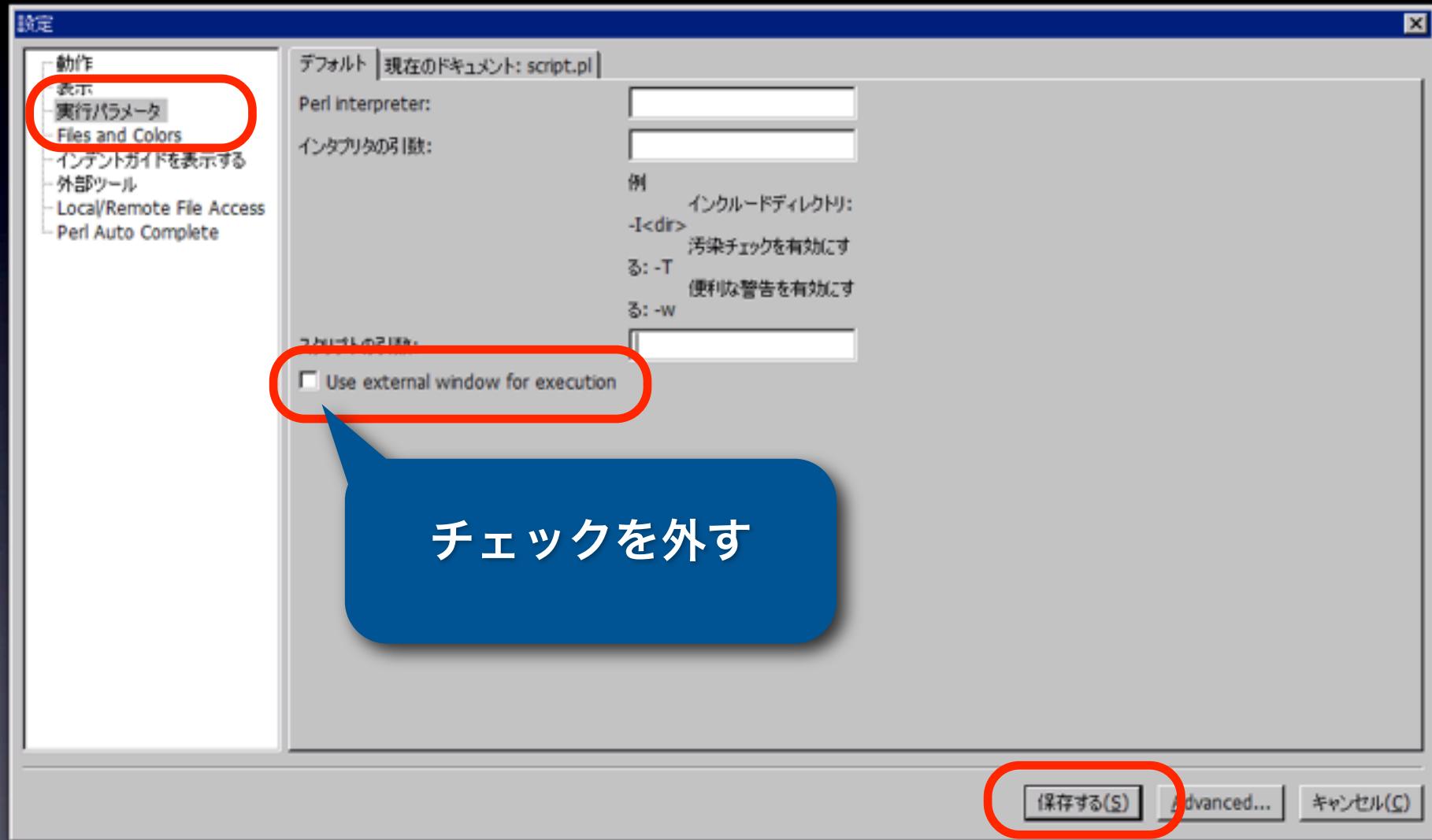
<http://padre.perlide.org>



まずは設定



まずは設定



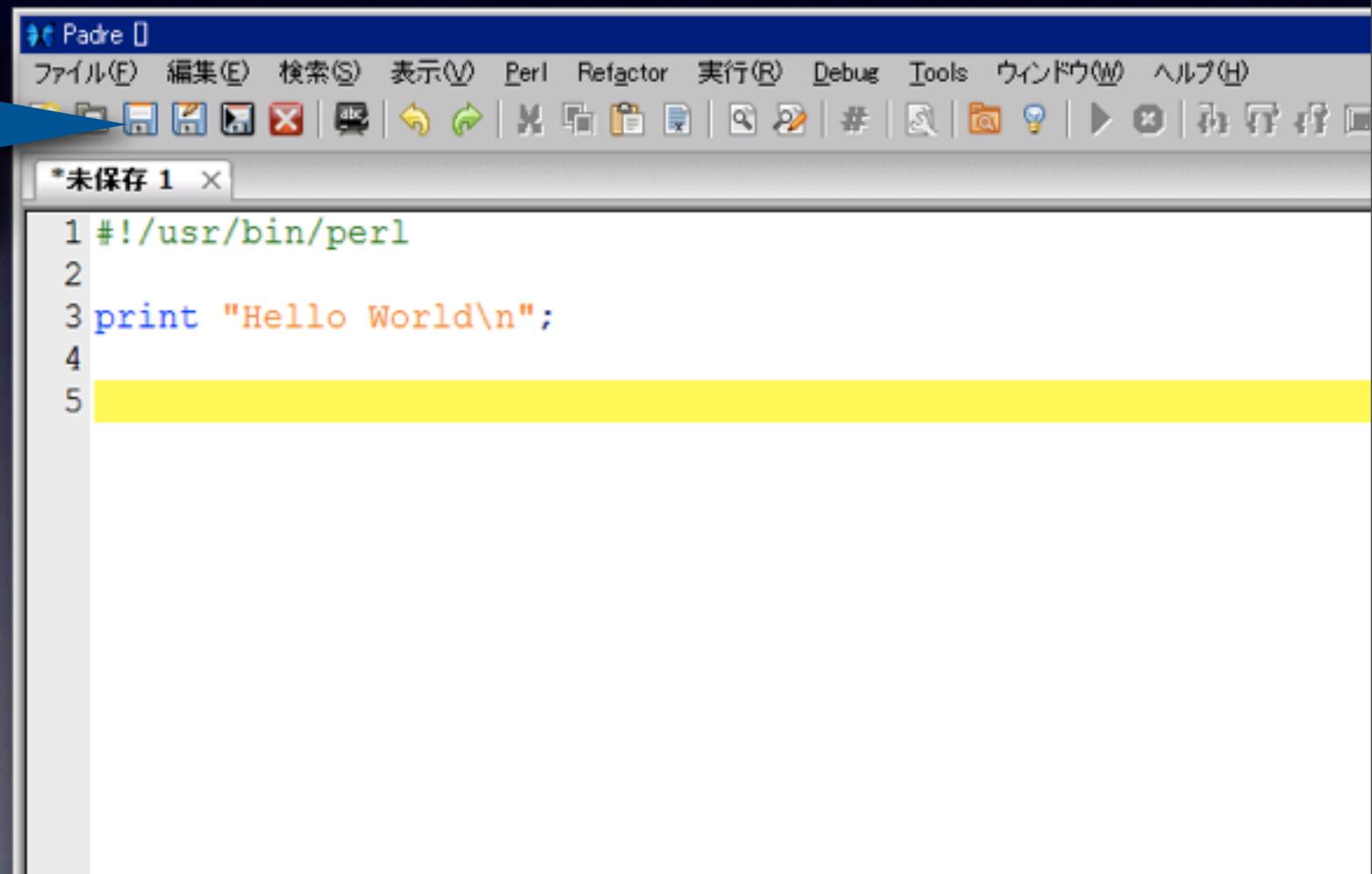
最初のプログラム

¥

```
#!/usr/bin/perl  
print "Hello World\n";
```

保存

(script.pl)

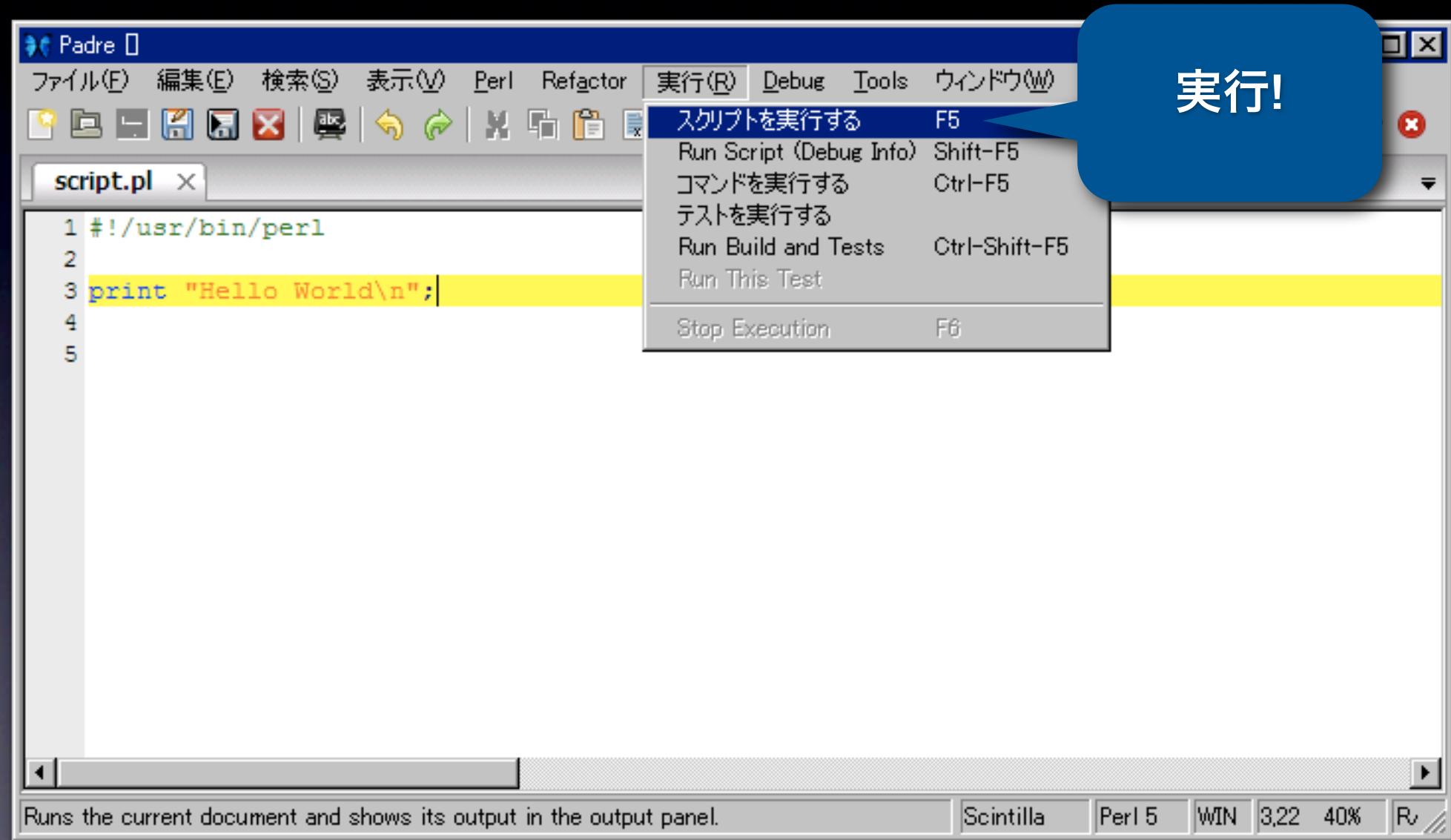


The screenshot shows the Padre IDE interface. The title bar reads "Padre" and the menu bar includes "ファイル(F)", "編集(E)", "検索(S)", "表示(V)", "Perl", "Refactor", "実行(R)", "Debug", "Tools", "ウインドウ(W)", and "ヘルプ(H)". The toolbar contains various icons for file operations. The main window displays a code editor with the following content:

```
*未保存 1 ×  
1 #!/usr/bin/perl  
2  
3 print "Hello World\n";  
4  
5
```

A yellow horizontal bar highlights the line "print \"Hello World\\n\";". A blue callout bubble on the left side points to the title bar, containing the text "保存 (script.pl)".

```
#!/usr/bin/perl  
print "Hello World\n";
```



```
#!/usr/bin/perl  
print "Hello World\n";
```

MacでもLinuxでも動くように

print文は「表示」を行う

改行させるために \n か ¥n をつける

Hello World と表示される

The screenshot shows a Perl development environment with the following components:

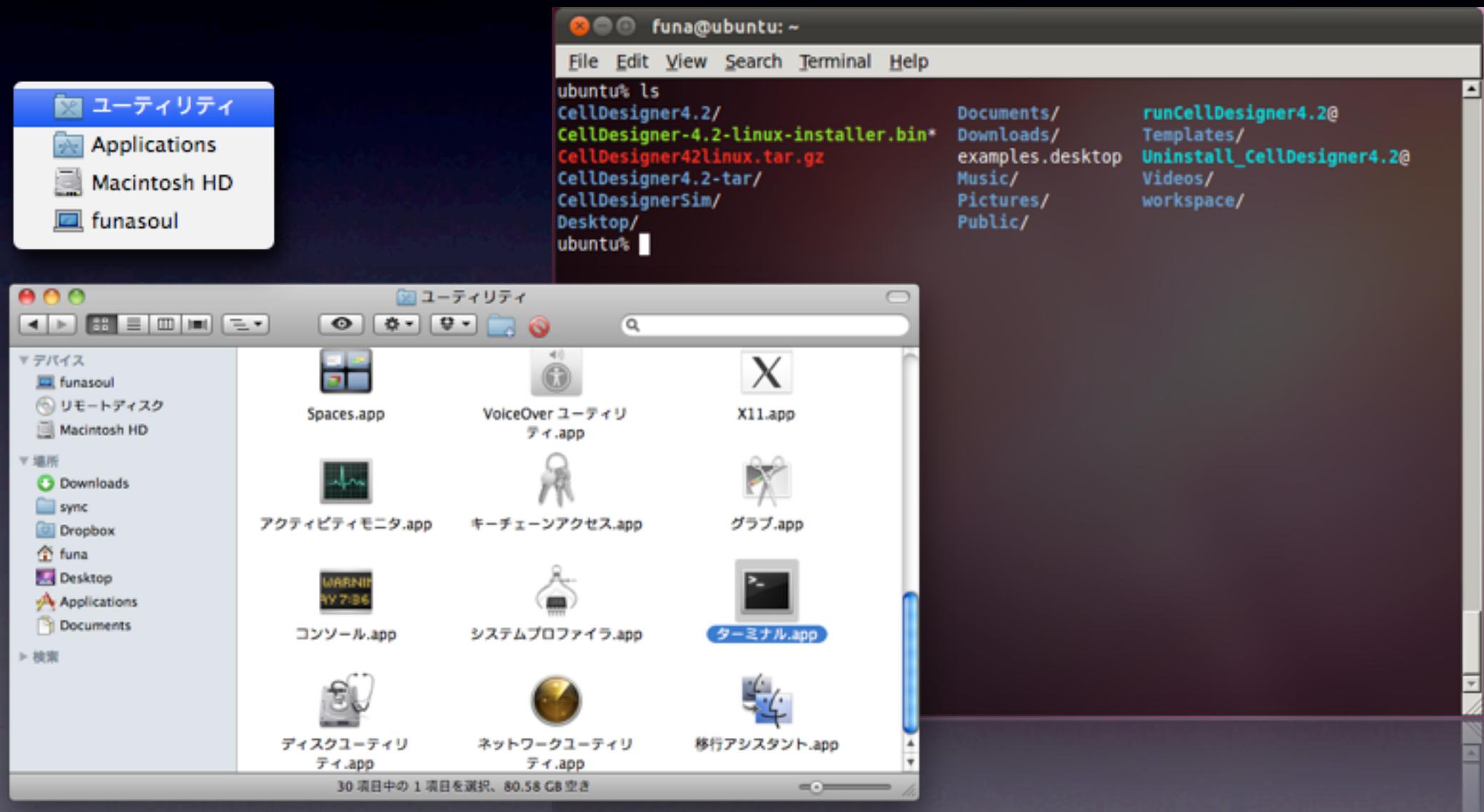
- Padre IDE:** A window titled "Padre" containing a file named "script.pl". The code in the editor is:

```
1 #!/usr/bin/perl  
2  
3 print "Hello World\n";  
4  
5
```

A yellow horizontal bar highlights the line "print "Hello World\n";".
- Output Window:** A window titled "出力" (Output) showing the text "Hello World" in red.
- Status Bar:** At the bottom, it displays "Scintilla" and "Perl 5" along with system information: "WIN 5.0 80% R".

Mac, Linuxの人

Terminalを起動



Emacs, Xcode, nanoなどで作成

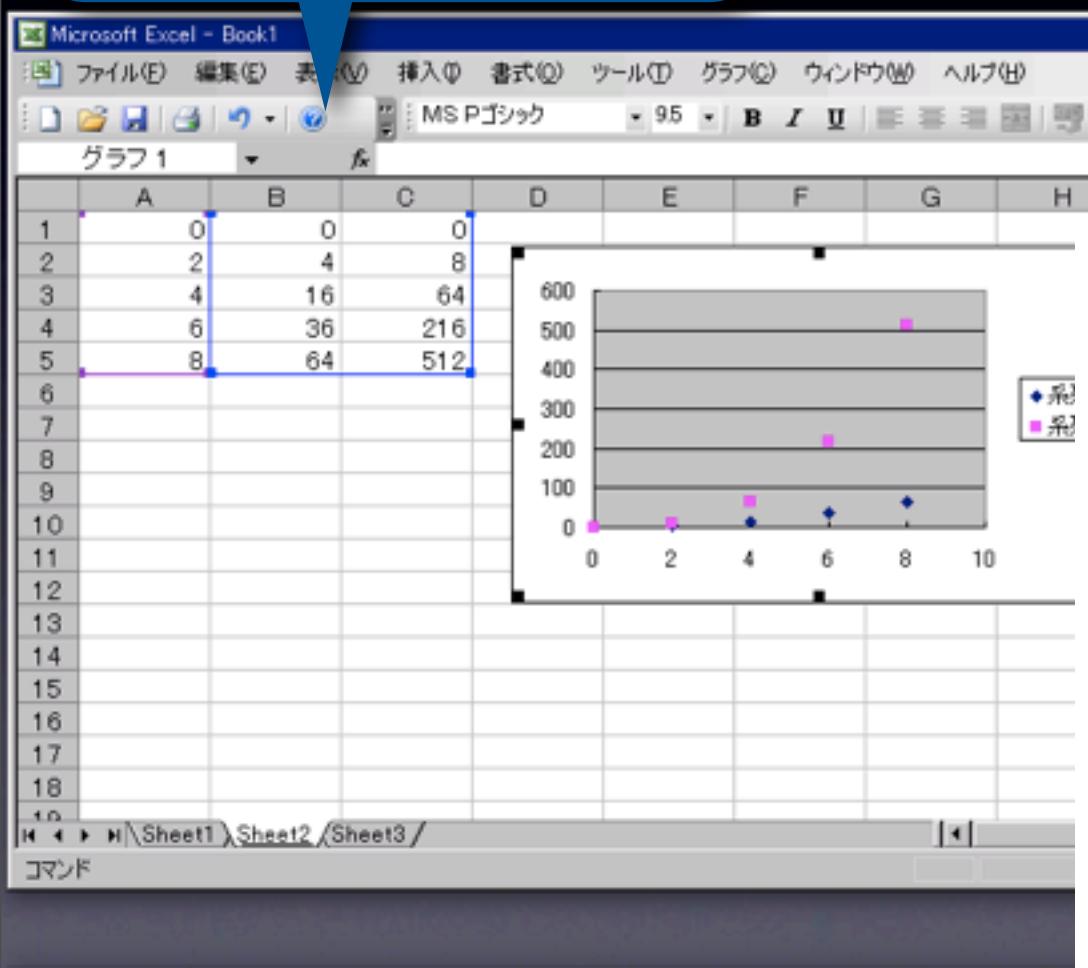
```
#!/usr/bin/perl  
print "Hello World\n";
```

emacs script.pl
(Perlのプログラムを書く)

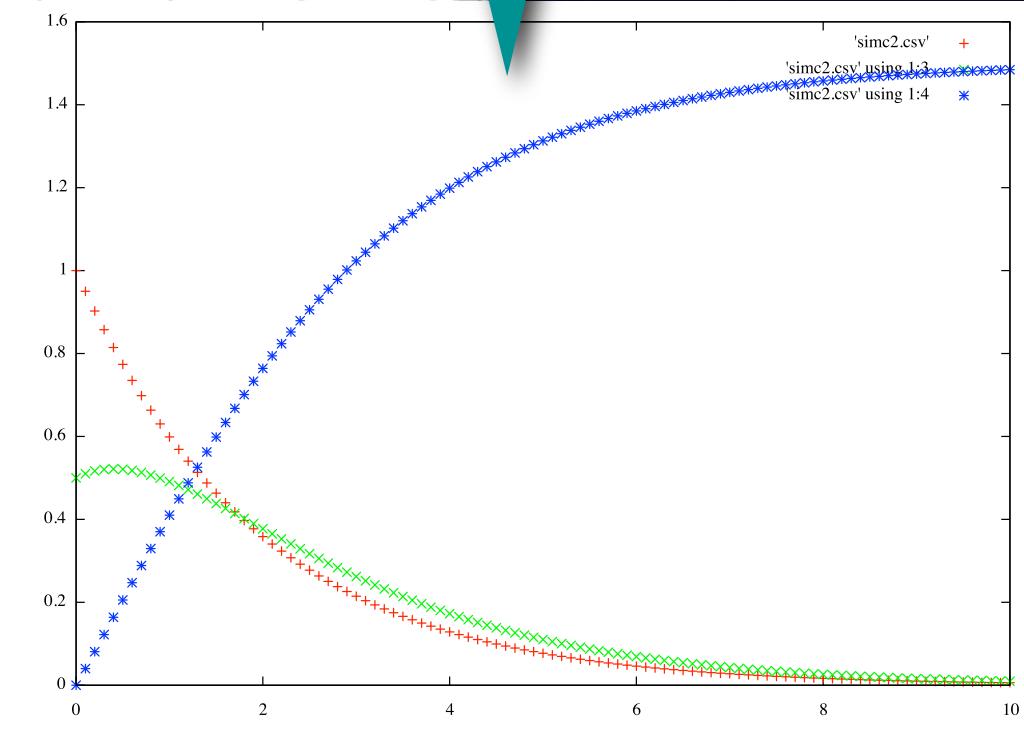
perl script.pl
Hello World

プロット

Excel等の表計算ソフト



gnuplot等



プロット用データを用意

```
#!/usr/bin/perl

print "0,0\n";
print "2,4\n";
print "4,16\n";
print "6,36\n";
print "8,64\n";
```

The screenshot shows the Padre IDE interface. The title bar says 'Padre'. The menu bar includes 'ファイル(F)', '編集(E)', '検索(S)', '表示(V)', 'Perl', 'Refactor', '実行(R)', 'Debug', and 'Tools'. The toolbar has various icons for file operations like new, open, save, and search. A tab bar at the top shows 'script.pl x'. The main code editor area contains the following Perl script:

```
1 #!/usr/bin/perl
2
3 print "0,0\n";
4 print "2,4\n";
5 print "4,16\n";
6 print "6,36\n";
7 print "8,64\n";
8
```

プロット用データを用意

```
#!/usr/bin/perl

print "0,0\n";
print "2,4\n";
print "4,16\n";
print "6,36\n";
print "8,64\n";
```

The screenshot shows the Padre IDE interface. On the left, a code editor window titled "script.pl" contains the Perl script provided above. On the right, an "Output" window displays the execution results:

```
0,0
2,4
4,16
6,36
8,64
```

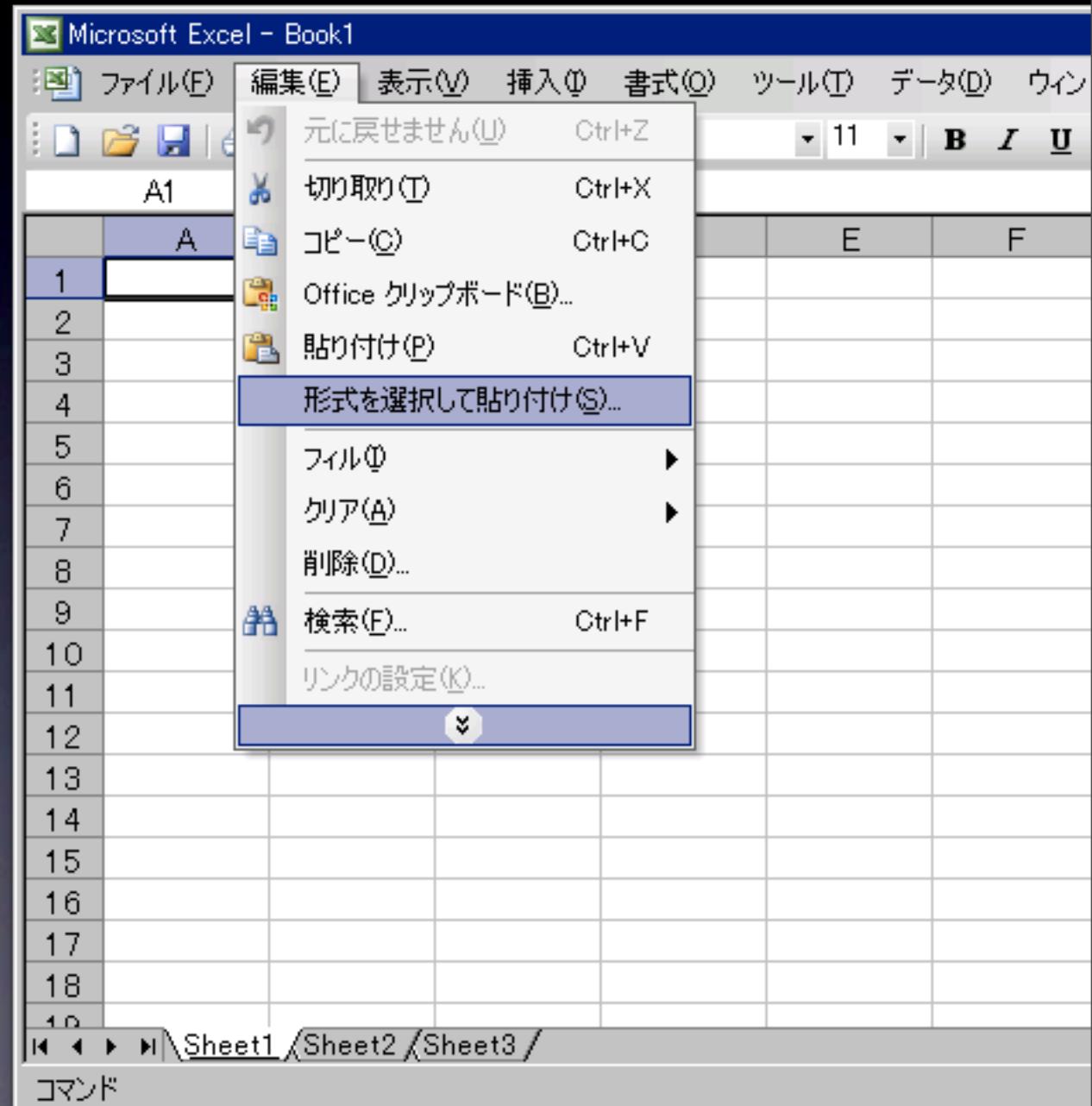
A blue callout bubble points to the context menu in the "Output" window, which is open over the first line of output. The menu items shown are:

- Undo Insert Text
- 切り直す(R)
- 切り取る(C)
- コピー(C)** (highlighted)
- 貼り付け(P)
- Delete

Text in the callout bubble says: "出力結果を全選択してコピー"

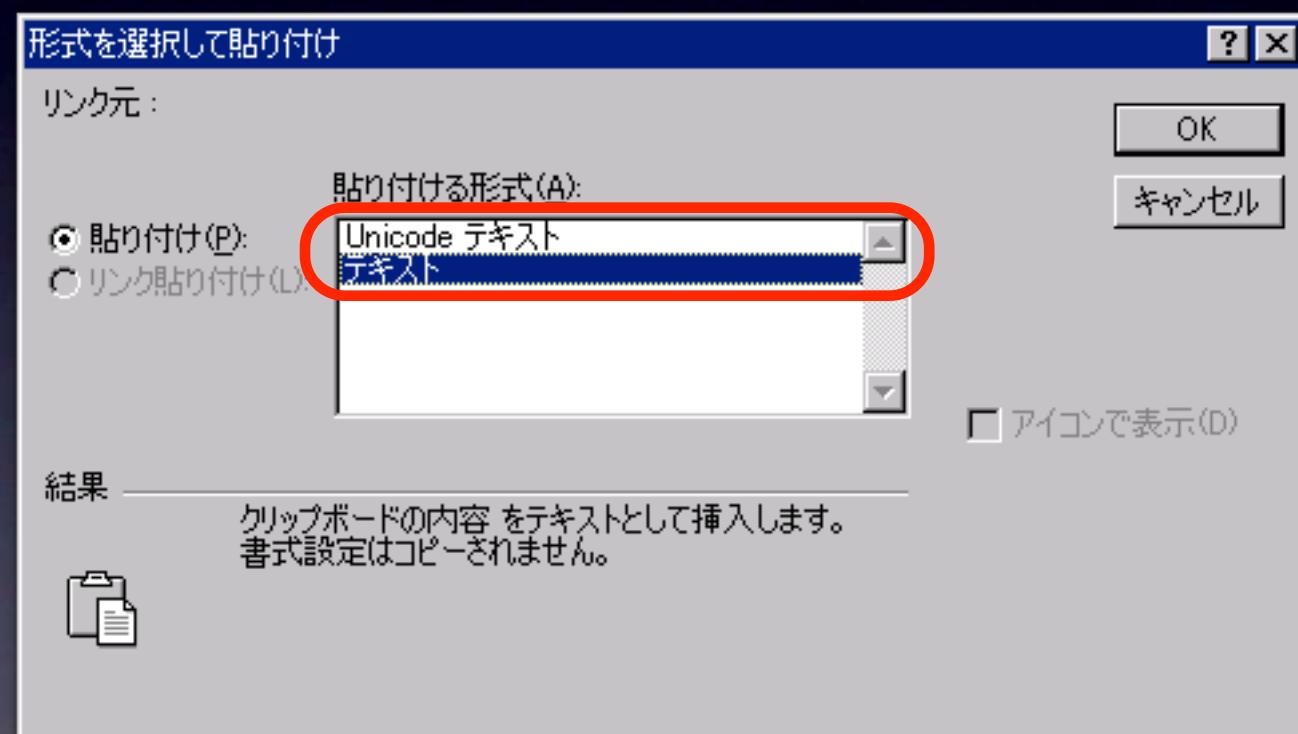
Excelに取り込む

```
#!/usr/bin/perl  
  
print "0,0\n";  
print "2,4\n";  
print "4,16\n";  
print "6,36\n";  
print "8,64\n";
```



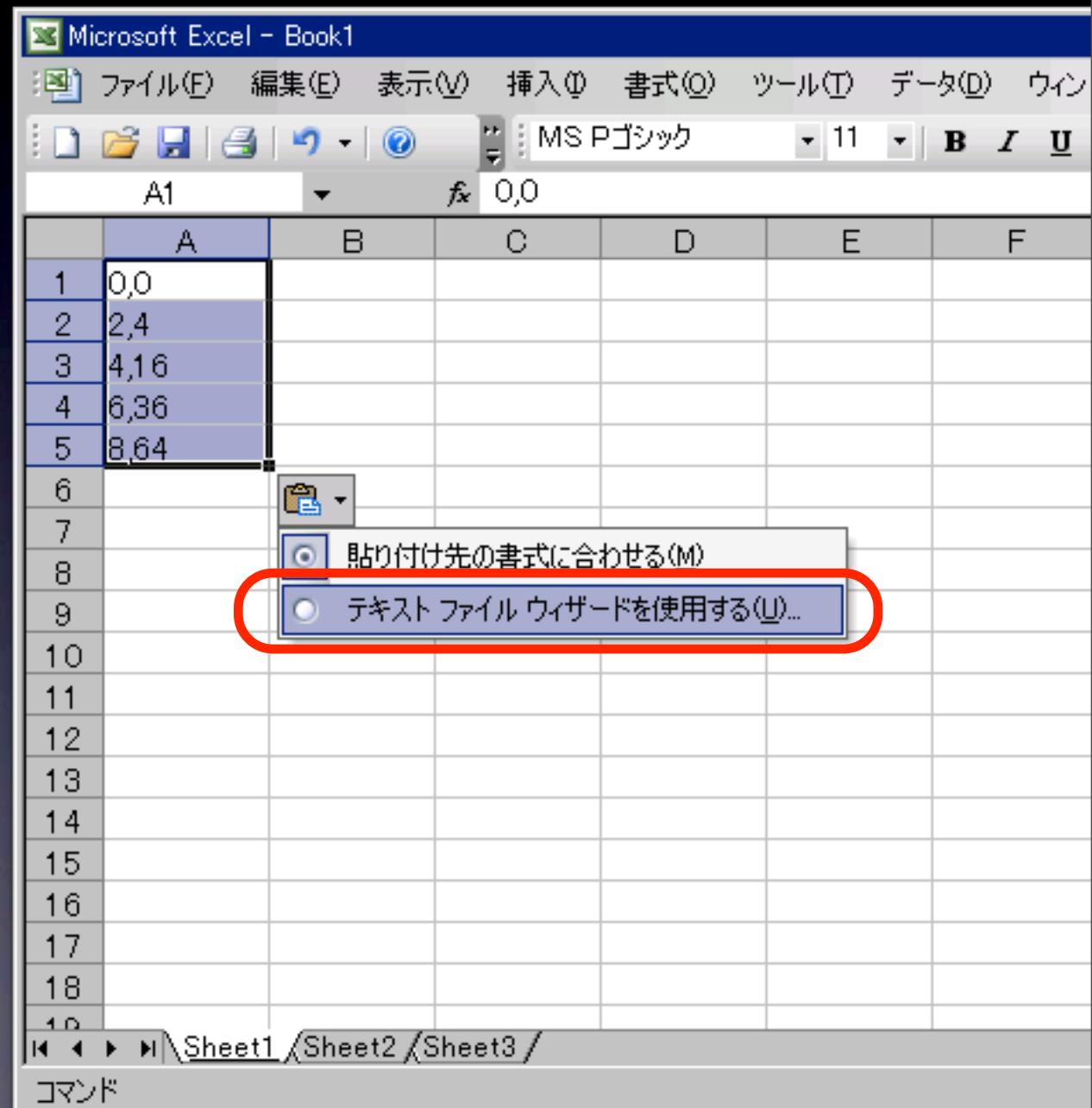
Excelに取り込む

```
#!/usr/bin/perl  
  
print "0,0\n";  
print "2,4\n";  
print "4,16\n";  
print "6,36\n";  
print "8,64\n";
```



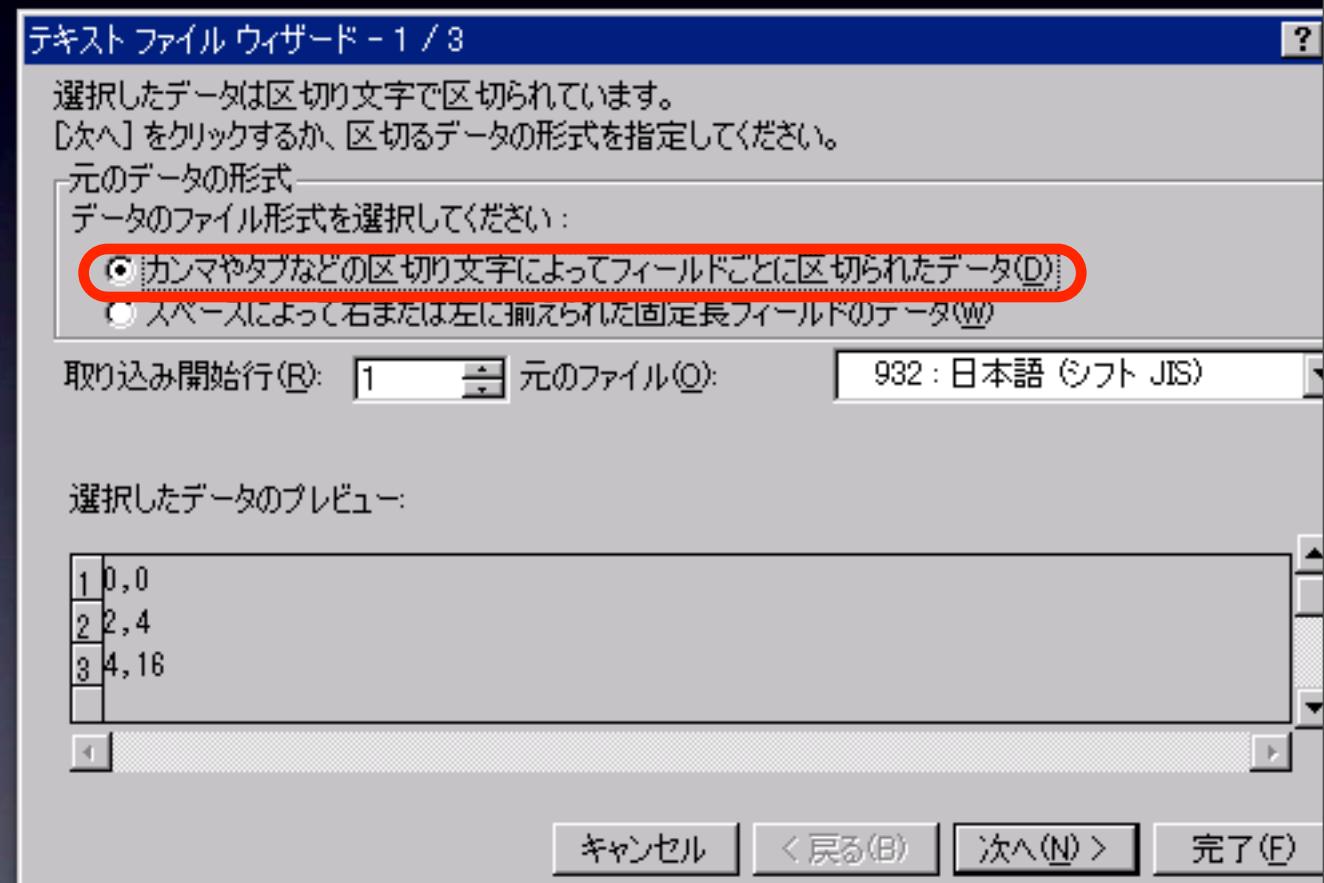
Excelに取り込む

```
#!/usr/bin/perl  
  
print "0,0\n";  
print "2,4\n";  
print "4,16\n";  
print "6,36\n";  
print "8,64\n";
```



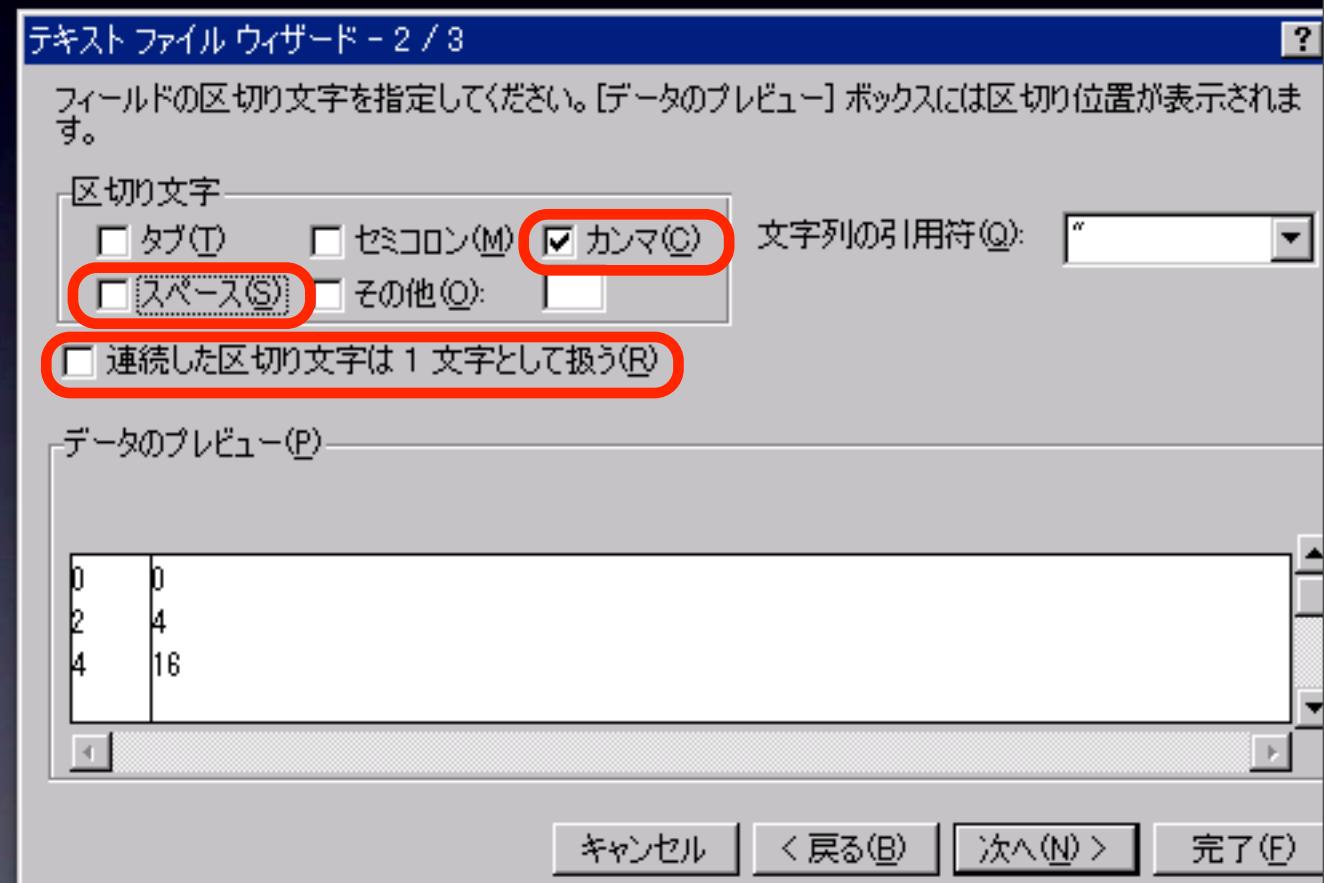
Excelに取り込む

```
#!/usr/bin/perl  
  
print "0,0\n";  
print "2,4\n";  
print "4,16\n";  
print "6,36\n";  
print "8,64\n";
```



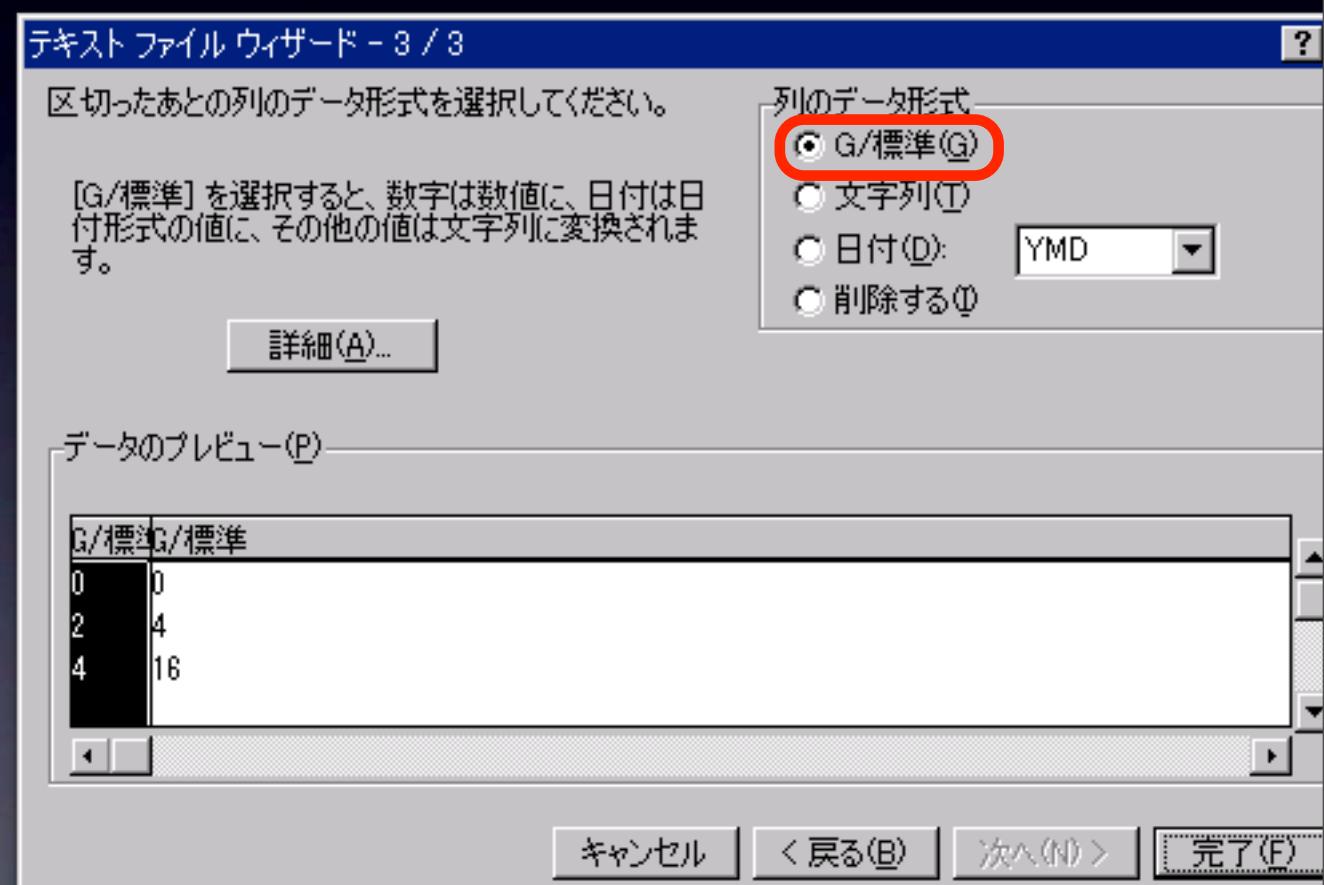
Excelに取り込む

```
#!/usr/bin/perl  
  
print "0,0\n";  
print "2,4\n";  
print "4,16\n";  
print "6,36\n";  
print "8,64\n";
```



Excelに取り込む

```
#!/usr/bin/perl  
  
print "0,0\n";  
print "2,4\n";  
print "4,16\n";  
print "6,36\n";  
print "8,64\n";
```

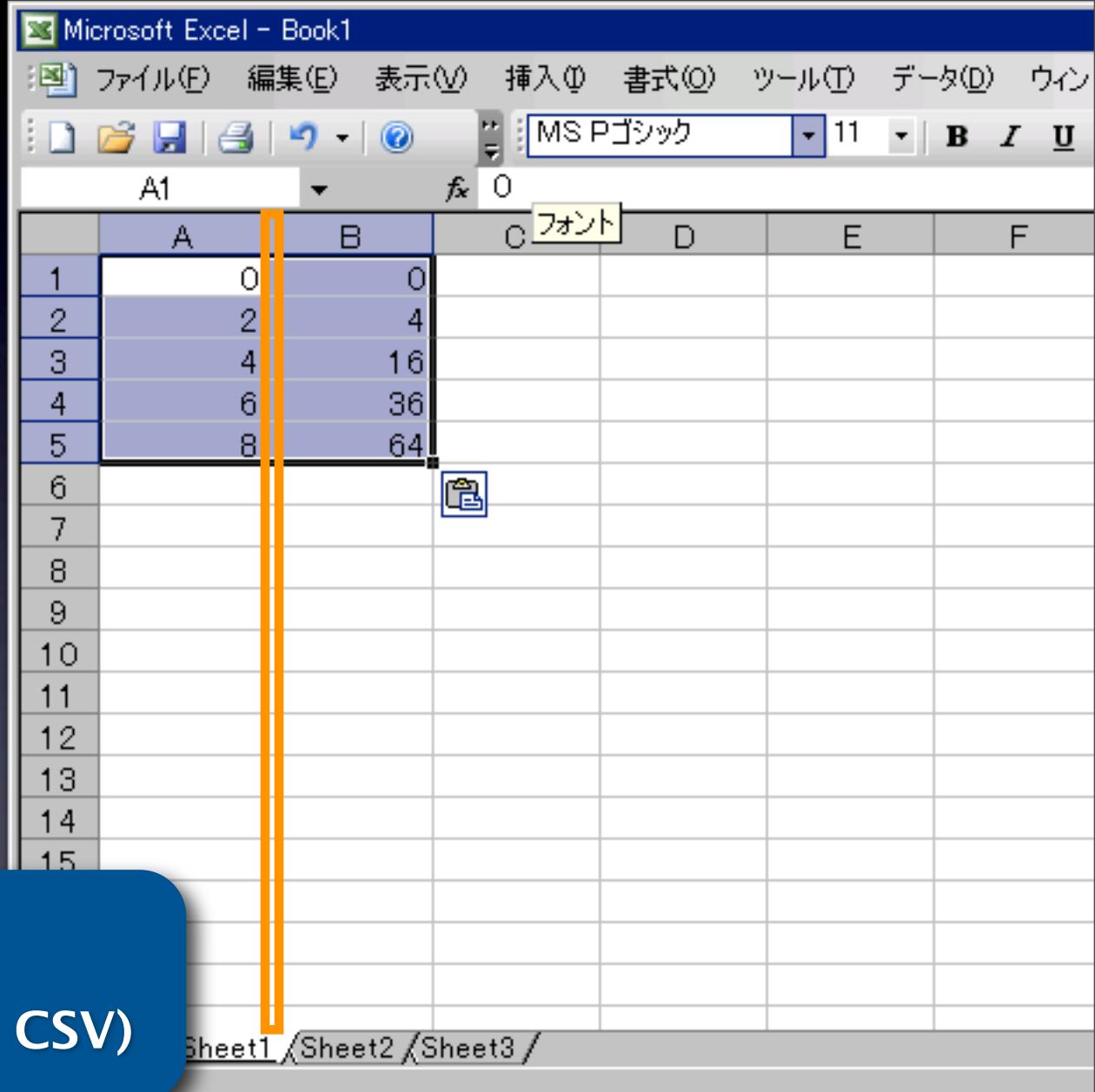


Excelに取り込む

```
#!/usr/bin/perl  
  
print "0,0\n";  
print "2,4\n";  
print "4,16\n";  
print "6,36\n";  
print "8,64\n";
```

カンマ区切り

(Comma Separated Values: CSV)

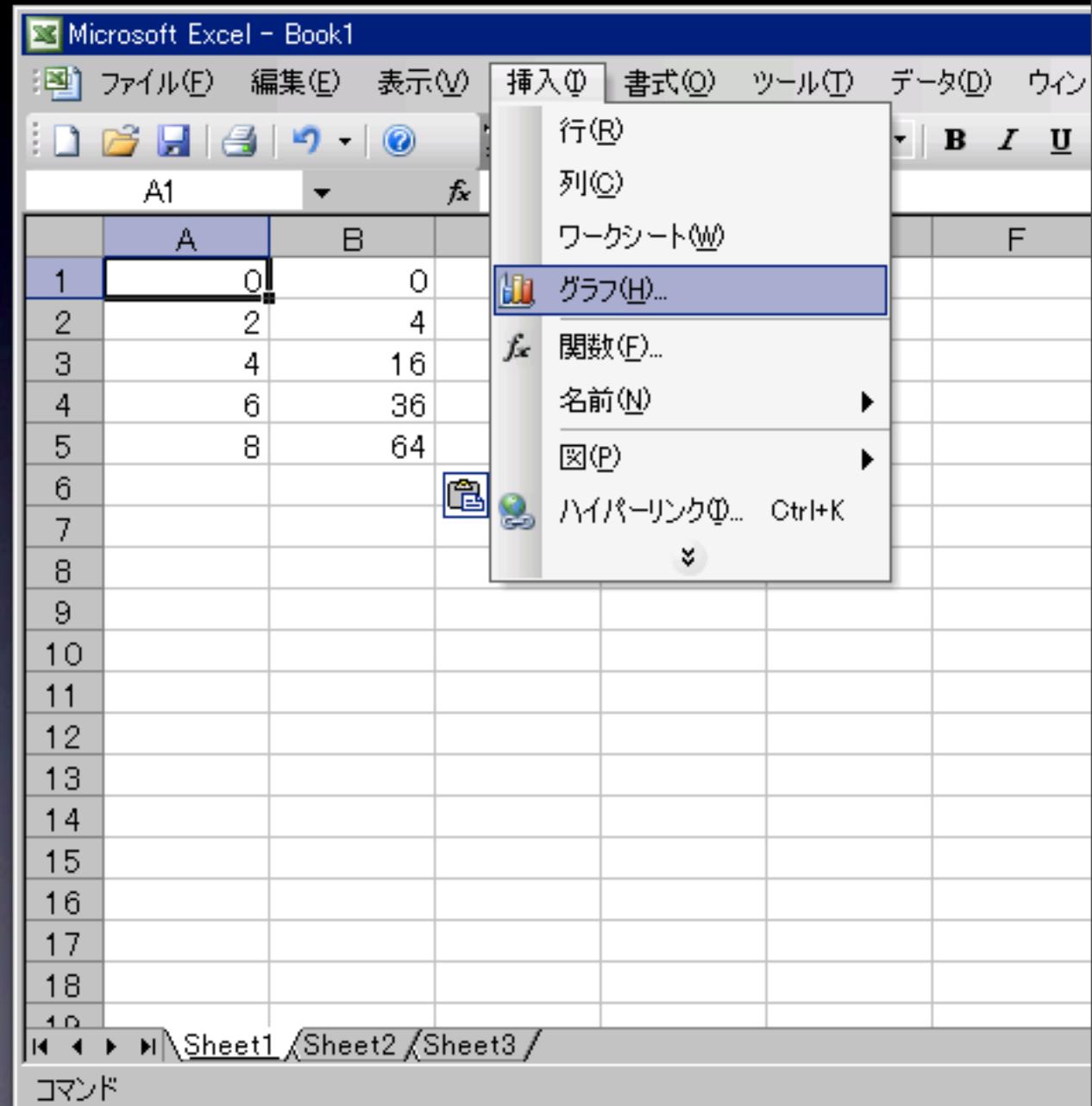


	A	B	C	D	E	F
1	0	0				
2	2	4				
3	4	16				
4	6	36				
5	8	64				
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						

Excelでプロット

```
#!/usr/bin/perl

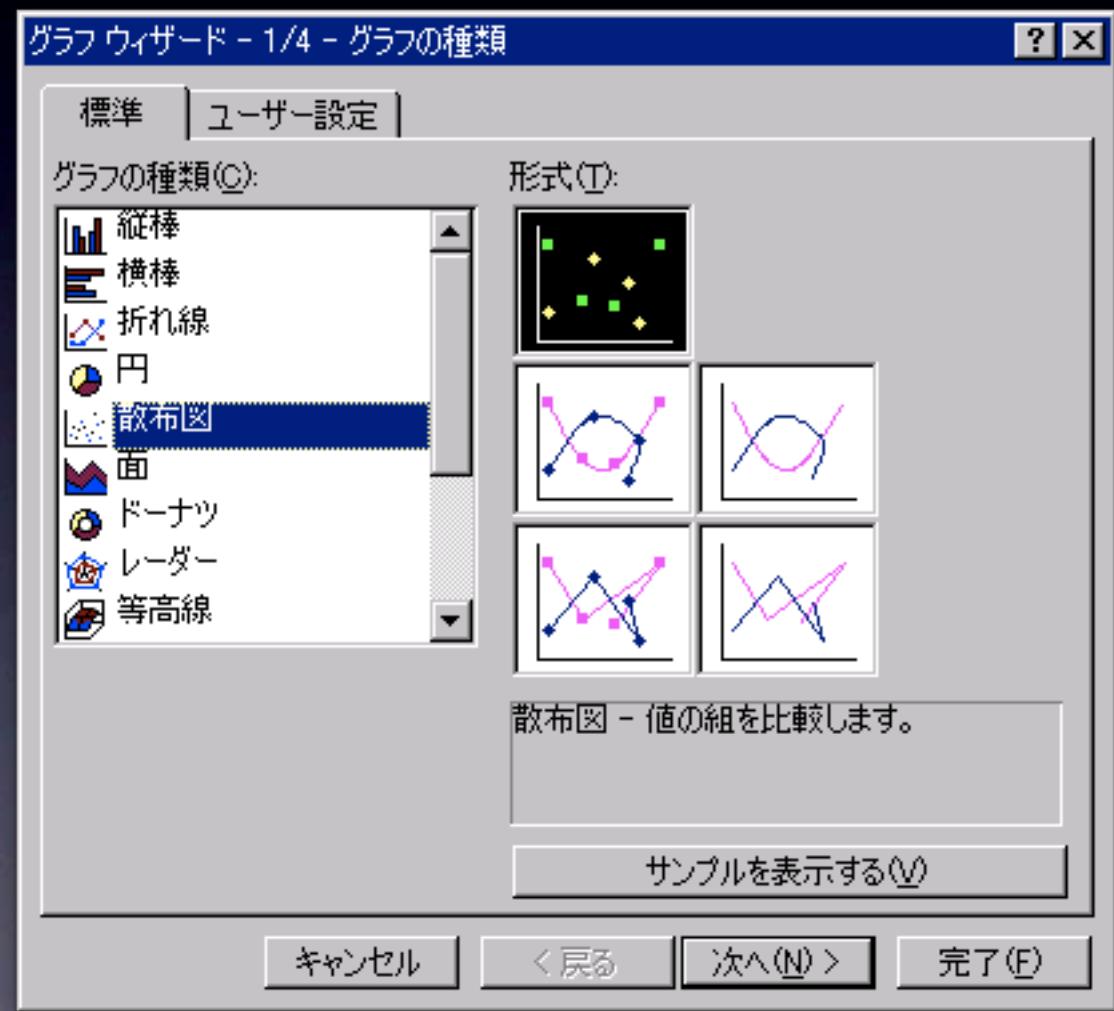
print "0,0\n";
print "2,4\n";
print "4,16\n";
print "6,36\n";
print "8,64\n";
```



Excelでプロット

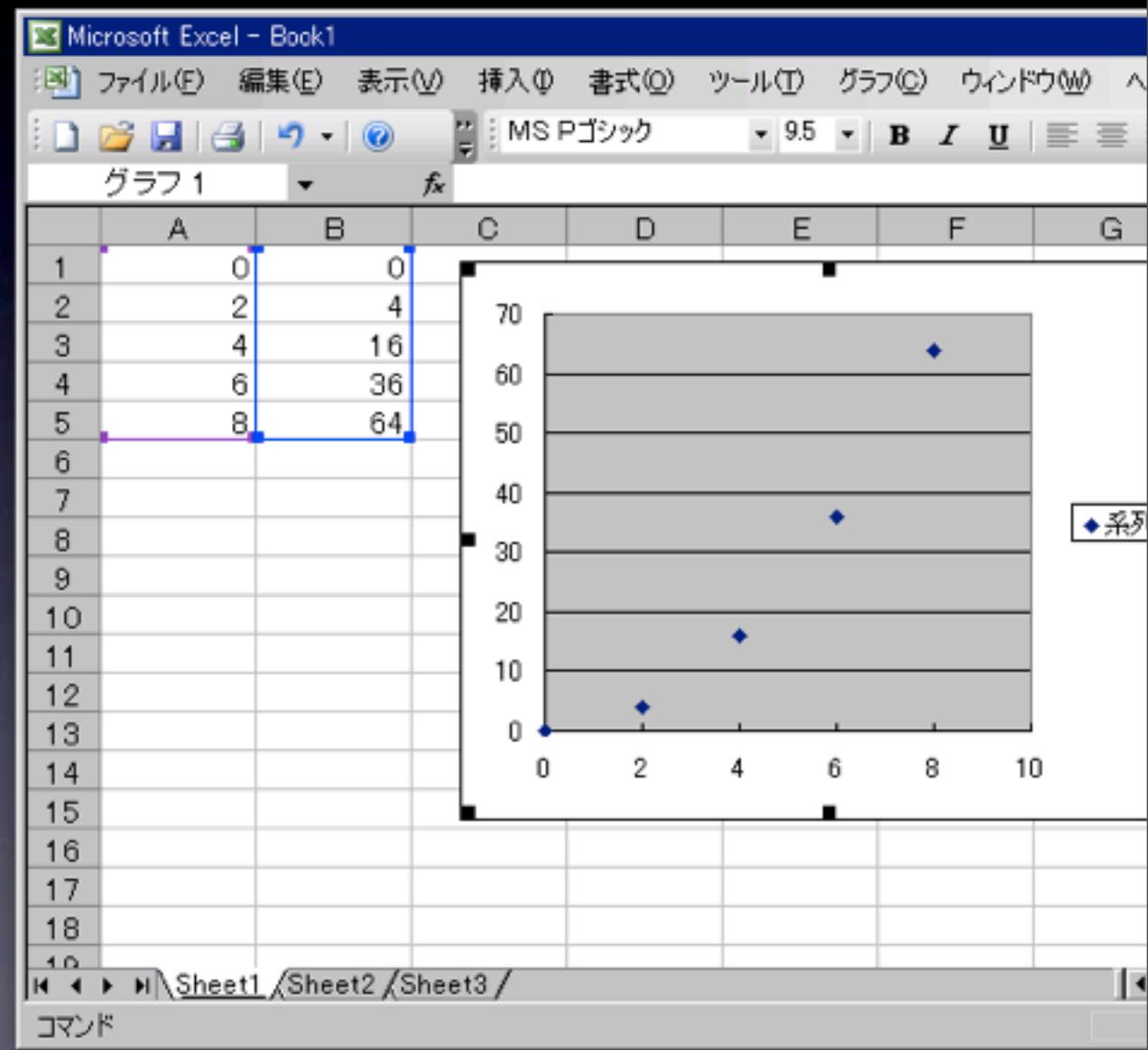
```
#!/usr/bin/perl

print "0,0\n";
print "2,4\n";
print "4,16\n";
print "6,36\n";
print "8,64\n";
```



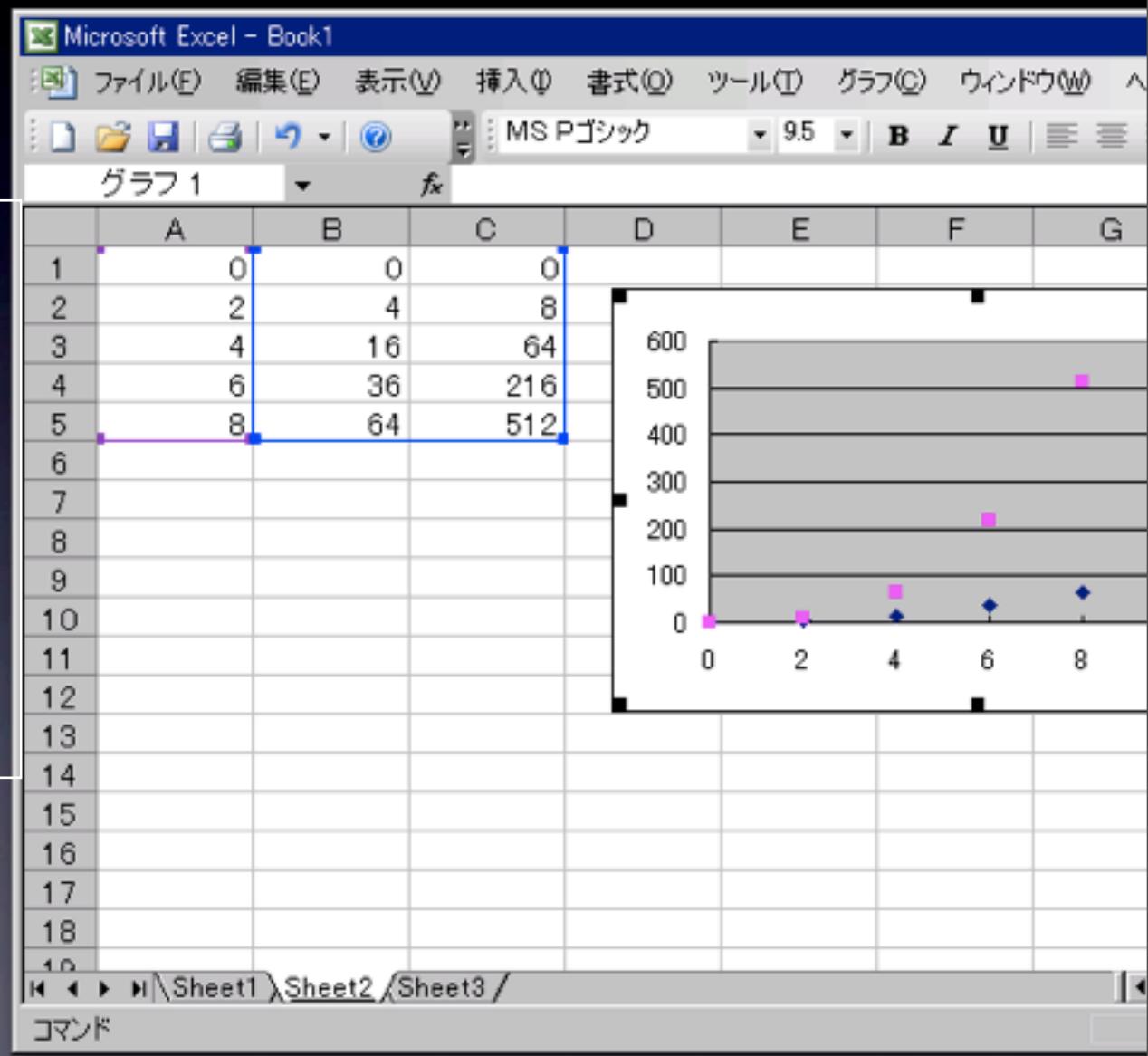
Excelでプロット

```
#!/usr/bin/perl  
  
print "0,0\n";  
print "2,4\n";  
print "4,16\n";  
print "6,36\n";  
print "8,64\n";
```



Excelでプロット

```
#!/usr/bin/perl  
  
print "0,0,0\n";  
print "2,4,8\n";  
print "4,16,64\n";  
print "6,36,216\n";  
print "8,64,512\n";
```



gnuplotでプロット

```
perl script.pl
```

0,0

2,4

4,16

6,36

8,64

とりあえず実行して確認

ファイル(data.csv)に保存

```
perl script.pl >! data.csv
```

gnuplot

gnuplot> set datafile separator ","

gnuplot> plot 'data.csv'

```
#!/usr/bin/perl  
  
print "0,0\n";  
print "2,4\n";  
print "4,16\n";  
print "6,36\n";  
print "8,64\n";
```

カンマ区切りだよ

gnuplotでプロット

```
perl script.pl >! data.csv  
gnuplot  
gnuplot> set datafile separator ","  
gnuplot> plot 'data.csv'  
gnuplot> replot 'data.csv' using 1:3
```

```
#!/usr/bin/perl  
  
print "0,0,0\n";  
print "2,4,8\n";  
print "4,16,64\n";  
print "6,36,216\n";  
print "8,64,512\n";
```

using 1:4
using 1:5

...

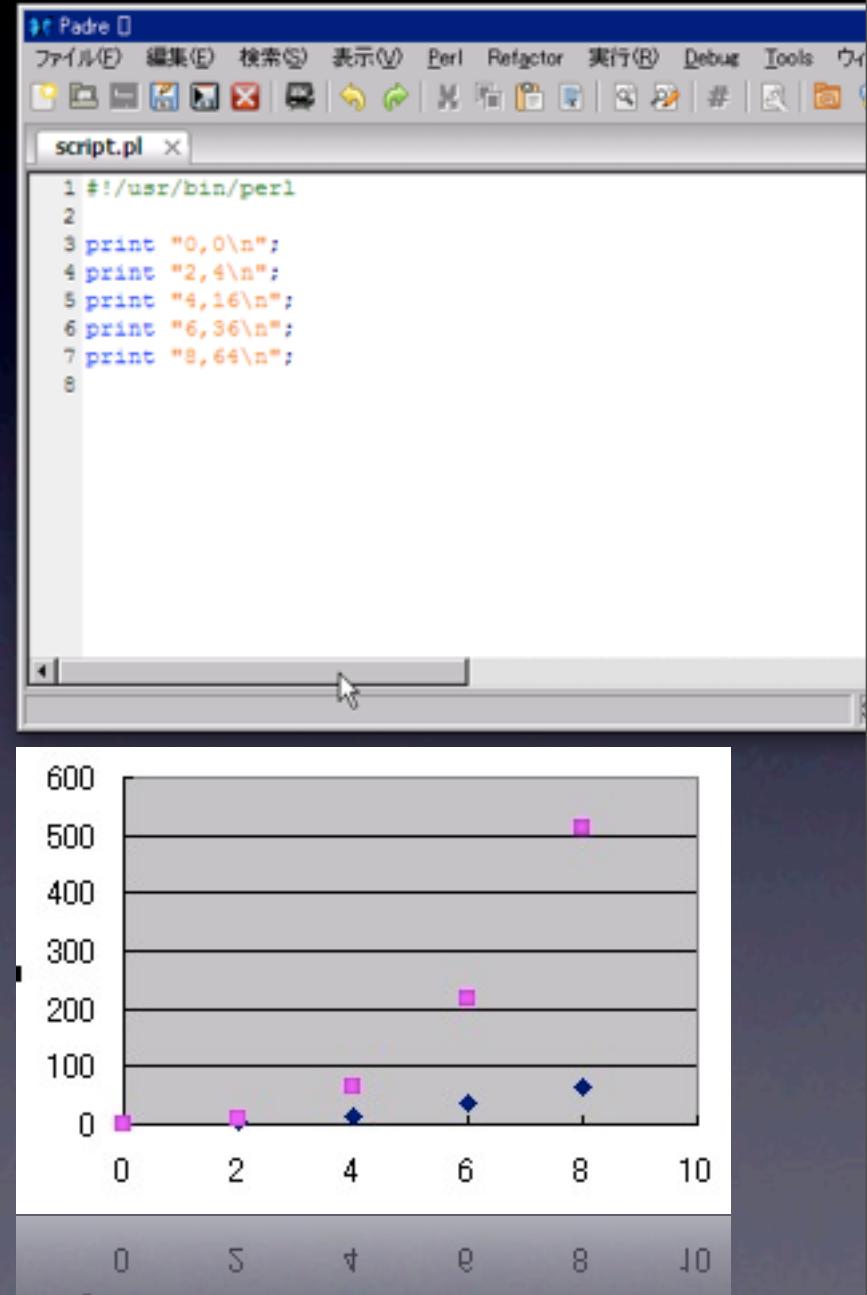
ここまでまとめ

- Perlでプログラムを書く

- Padre, Emacs等
- print文で表示
- CSV(カンマ区切り)で表示

- プロット

- CSVを読み込む
- Excel, gnuplot

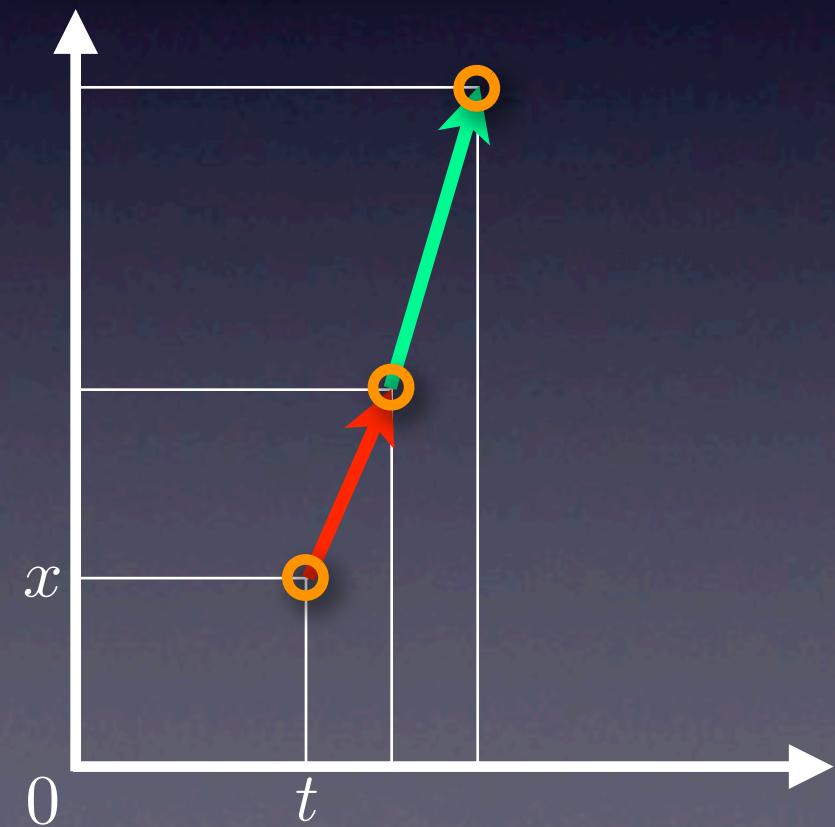


足りないもの

- Perlでプログラムを書く

数値積分

- Padre, Emacs等
- print文で表示
- CSV(カンマ区切り)で表示
- プロット
- CSVを読み込む
- Excel, gnuplot



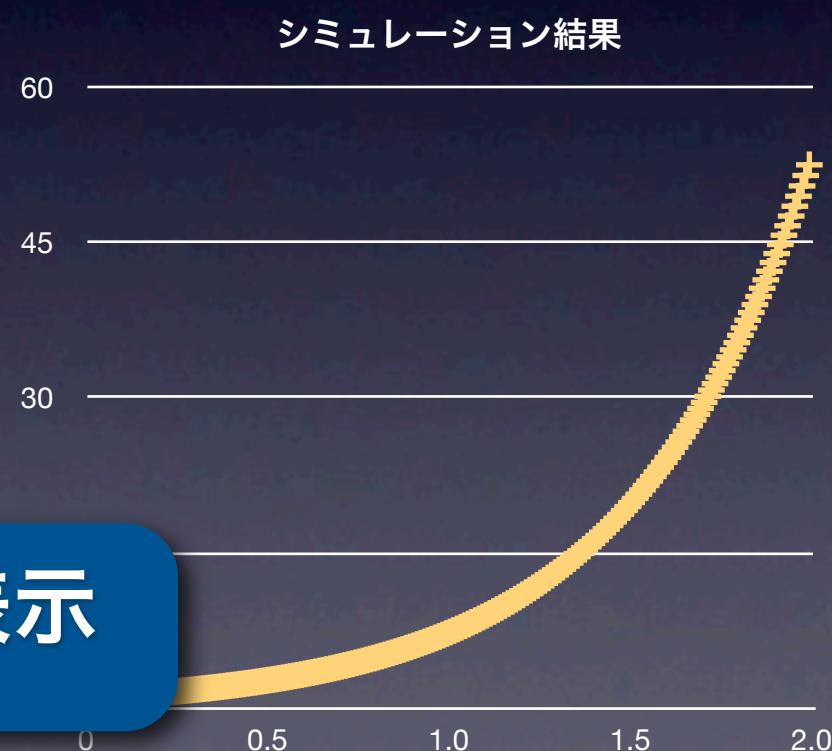
ODE Simulator

- $\frac{dx}{dt} = 2x$ を解くシミュレータ $t = 0, x = 1.0$

```
#!/usr/bin/perl
```

```
$dt = 0.01;  
$t = 0.0;  
$x = 1.0;  
  
for ($i = 0; $i <= 200; $i++) {  
    print "$t,$x\n";  
    $dx = 2 * $x * $dt;  
    $x = $x + $dx;  
    $t = $t + $dt;  
}
```

CSVで表示



ODE Simulator

- $\frac{dx}{dt} = 2x$ を解くシミュレータ $t = 0, x = 1.0$

```
#!/usr/bin/perl
```

Δt の値

```
$dt = 0.01;  
$t = 0.0;  
$x = 1.0;
```

x, t の初期値

```
for ($i = 0; $i <= 200; $i++) {
```

201回繰り返す

```
    print "$t,$x\n";  
    $dx = 2 * $x * $dt;  
    $x = $x + $dx;  
    $t = $t + $dt;
```

繰り返しの内容

```
}
```

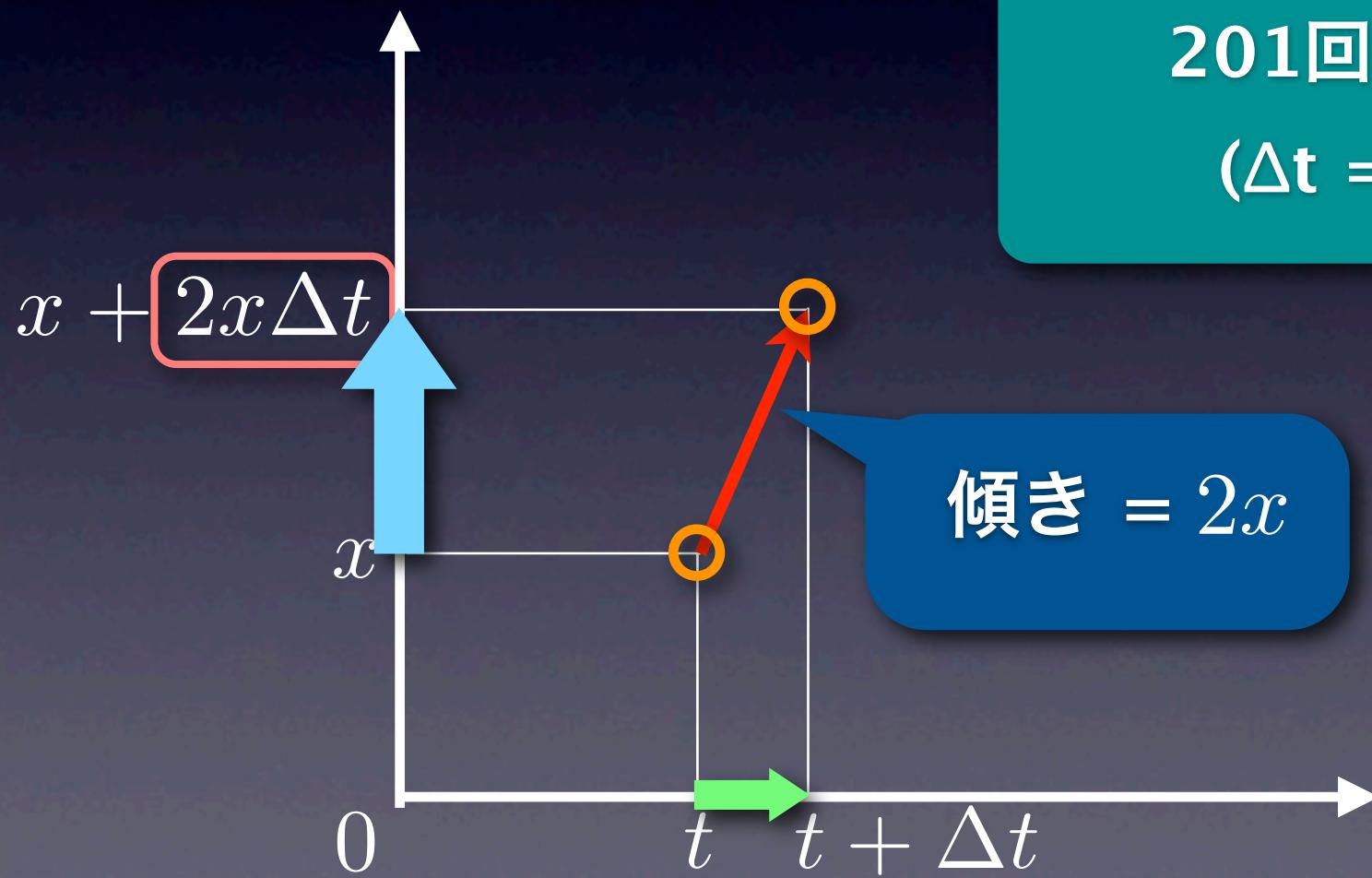
```

for ($i = 0; $i <= 200; $i++) {
    print "$t,$x\n";
    $dx = 2 * $x * $dt;
    $x = $x + $dx;
    $t = $t + $dt;
}

```

$$\frac{dx}{dt} = 2x$$

$t=0$ から $t=2.0$ まで
201回繰り返す
($\Delta t = 0.01$)



ポイント

- $\frac{dx}{dt} = 2x$ を解くシミュレータ $t = 0, x = 1.0$

```
#!/usr/bin/perl
```

```
$dt = 0.01;  
$t = 0.0;  
$x = 1.0;
```

Δt の値

x, t の初期値

```
for ($i = 0; $i <= 200; $i++) {  
    print "$t,$x\n";  
    $dx = 2 * $x * $dt;  
    $x = $x + $dx;  
    $t = $t + $dt;  
}
```

シミュレーション時間
と Δt によって繰り返し
回数を決める

$dx/dt =$ の右辺を書く

x と t をちょっと増やす(減らす)

Padre

ファイル(F) 編集(E) 検索(S) 表示(V) Perl Refactor 実行(R) Debug Tools ウィンドウ(W) ヘルプ(H)

script.pl simulate.pl X

```
1 #!/usr/bin/perl
2
3 $dt = 0.01;
4 $t = 0.0;
5 $x = 1.0;
6
7 for ($i = 0; $i <= 200; $i++) {
8     print "$t,$x\n";
9     $dx = 2 * $x * $dt;
10    $x = $x + $dx;
11    $t = $t + $dt
12 }
13
```

Scintilla Perl 5 WIN 13,0 92% R

Padre

ファイル(F) 編集(E) 検索(S) 表示(V) Perl Refactor 実行(R) Debug Tools ウィンドウ(W) ヘルプ(H)

script.pl simulate.pl

```
1 #!/usr/bin/perl
2
3 $dt = 0.01;
4 $t = 0.0;
5 $x = 1.0;
6
7 for ($i = 0; $i <= 200; $i++) {
8     print "$t,$x\n";
9     $dx = 2 * $x * $dt;
10    $x = $x + $dx;
11    $t = $t + $dt
12 }
13
```

出力

1.97,49.457691026366
1.98,50.446844846894
1.99,51.455781743832
2.52.4848973787087

Undo Insert Text Ctrl+Z
切り直す(R) Ctrl+Y
切り取る(T)
コピー(C)
貼り付け(P)

C:\#Documents and Setting

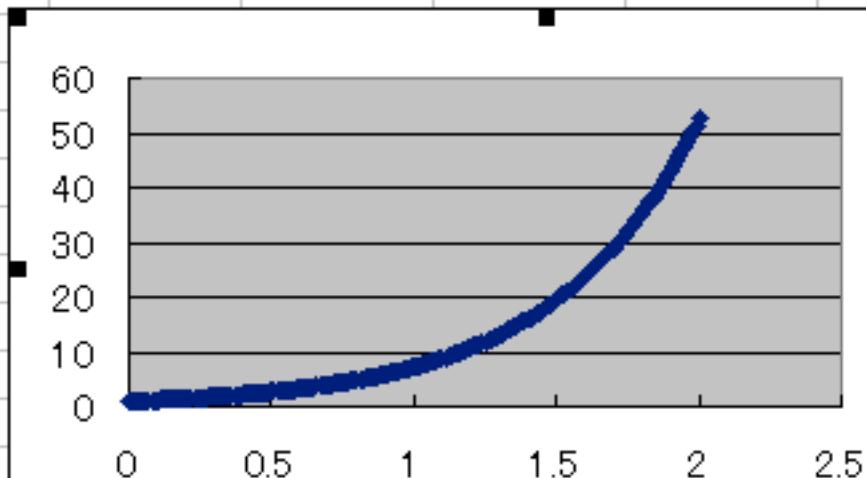
simulate.pl Scintilla Perl 5 WIN 13.0 92% R



グラフ エリア



	A	B	C	D	E	F	G	H	I	J
1		0	1							
2	0.01		1.02							
3	0.02		1.0404							
4	0.03		1.061208							
5	0.04		1.082432							
6	0.05		1.104081							
7	0.06		1.126162							
8	0.07		1.148686							
9	0.08		1.171659							
10	0.09		1.195093							
11	0.1		1.218994							
12	0.11		1.243374							
13	0.12		1.268242							
14	0.13		1.293607							
15	0.14		1.319479							
16	0.15		1.345868							
17	0.16		1.372786							
18	0.17		1.400241							
19	0.18		1.428246							



Mac, Linuxの人

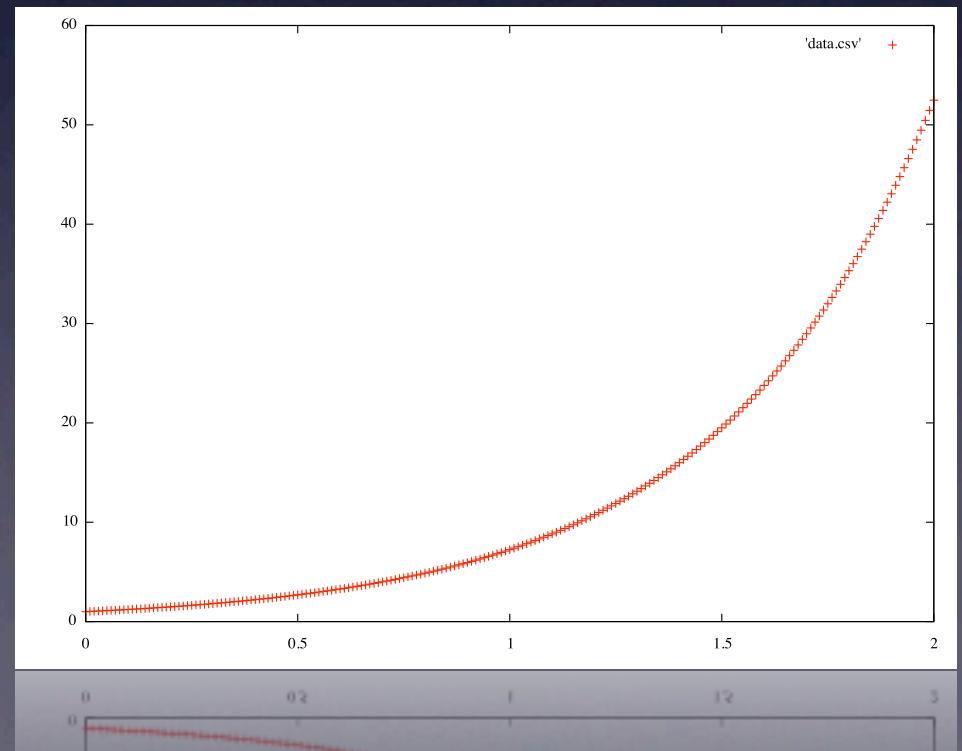
```
emacs simulate.pl
```

```
perl simulate.pl >! data.csv
```

```
gnuplot
```

```
gnuplot> set datafile separator ","
```

```
gnuplot> plot 'data.csv'
```



C言語版

- $\frac{dx}{dt} = 2x$ を解くシミュレータ $t = 0, x = 1.0$

```
#include<stdio.h>

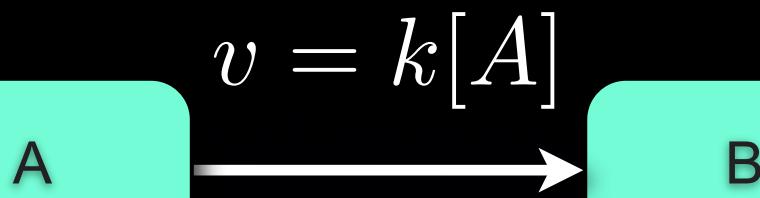
int main(void) {
    int i;
    double dx;
    double dt = 0.01;
    double t = 0.0;
    double x = 1.0;

    for (i = 0; i <= 200; i++) {
        printf("%lf,%lf\n", t, x);
        dx = 2.0 * x * dt;
        x = x + dx;
        t = t + dt;
    }
    return 0;
}
```

emacs simulate.c
gcc -Wall simulate.c
.a.out >! data.csv
(プロットはPerl版と同様)

Cコンパイラは gcc,
MSVC, LLVM Clang 等

2変数モデル



```
#!/usr/bin/perl
```

```
$dt = 0.01;  
$t = 0.0;  
$A = 1.0;  
$B = 0.0;  
$k = 1.5;
```

```
for ($i = 0; $i <= 200; $i++) {  
    print "$t, $A, $B\n";  
    $dA = - $k * $A * $dt;  
    $dB = $k * $A * $dt;  
    $A = $A + $dA;  
    $B = $B + $dB;  
    $t = $t + $dt  
}
```

$$\frac{d[A]}{dt} = -k[A]$$
$$\frac{d[B]}{dt} = k[A]$$

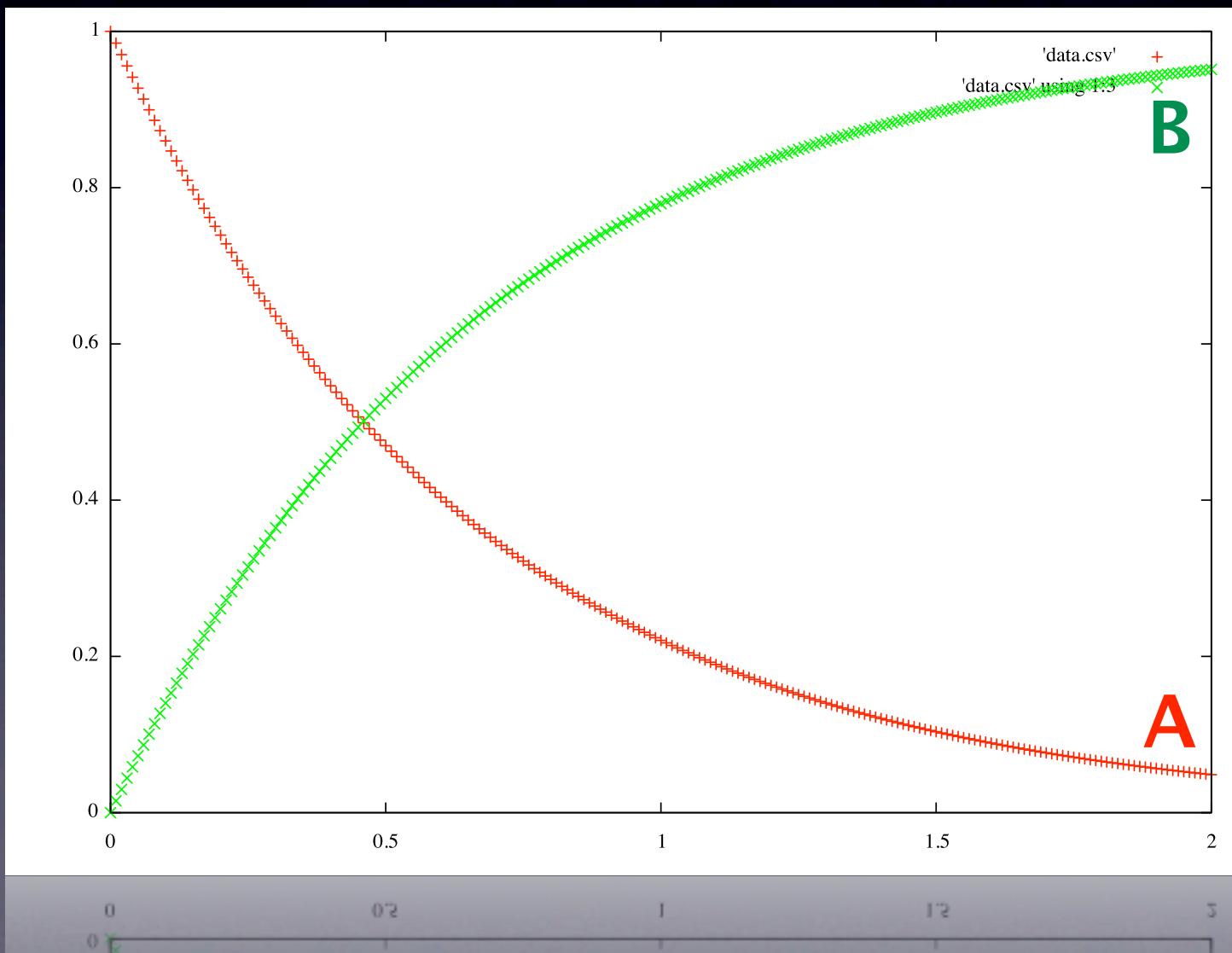
$$[A]_0 = 1.0$$

$$[B]_0 = 0.0$$

$$k = 1.5$$

A, B, t をちょっと増やす(減らす)

2変数モデル



C言語版

```
#include<stdio.h>

int main(void) {
    int i;
    double dA, dB;
    double dt = 0.01;
    double t = 0.0;
    double A = 1.0;
    double B = 0.0;
    double k = 1.5;

    for (i = 0; i <= 200; i++) {
        printf("%lf,%lf,%lf\n", t, A, B);
        dA = - k * A * dt;
        dB = k * A * dt;
        A = A + dA;
        B = B + dB;
        t = t + dt;
    }
    return 0;
}
```

```
emacs simulate.c
gcc -Wall simulate.c
./a.out >! data.csv
(プロットはPerl版と同様)
```

3変数モデル

$$v_1 = k_1[A] \quad v_2 = k_2[B]$$



$$\frac{d[A]}{dt} = -v_1 \quad \frac{d[B]}{dt} = v_1 - v_2 \quad \frac{d[C]}{dt} = v_2$$

$$[A]_0 = 1.0, \quad [B]_0 = 0.5, \quad [C]_0 = 0.0$$

$$k_1 = 0.5, \quad k_2 = 0.8$$

```
#!/usr/bin/perl
```

```
$dt = 0.1;  
$t = 0.0;  
$A = 1.0;  
$B = 0.5;  
$C = 0.0;  
$k1 = 0.5;  
$k2 = 0.8;
```

```
for ($i = 0; $i <= 100; $i++) {  
    print "$t,$A,$B,$C\n";  
    $v1 = $k1 * $A;  
    $v2 = $k2 * $B;  
    $dA = - $v1 * $dt;  
    $dB = $v1 * $dt - $v2 * $dt;  
    $dC = $v2 * $dt;  
    $A = $A + $dA;  
    $B = $B + $dB;  
    $C = $C + $dC;  
    $t = $t + $dt  
}
```

Perl版

$$v_1 = k_1[A]$$

$$v_2 = k_2[B]$$

$$\frac{d[A]}{dt} = -v_1$$

$$\frac{d[B]}{dt} = v_1 - v_2$$

$$\frac{d[C]}{dt} = v_2$$

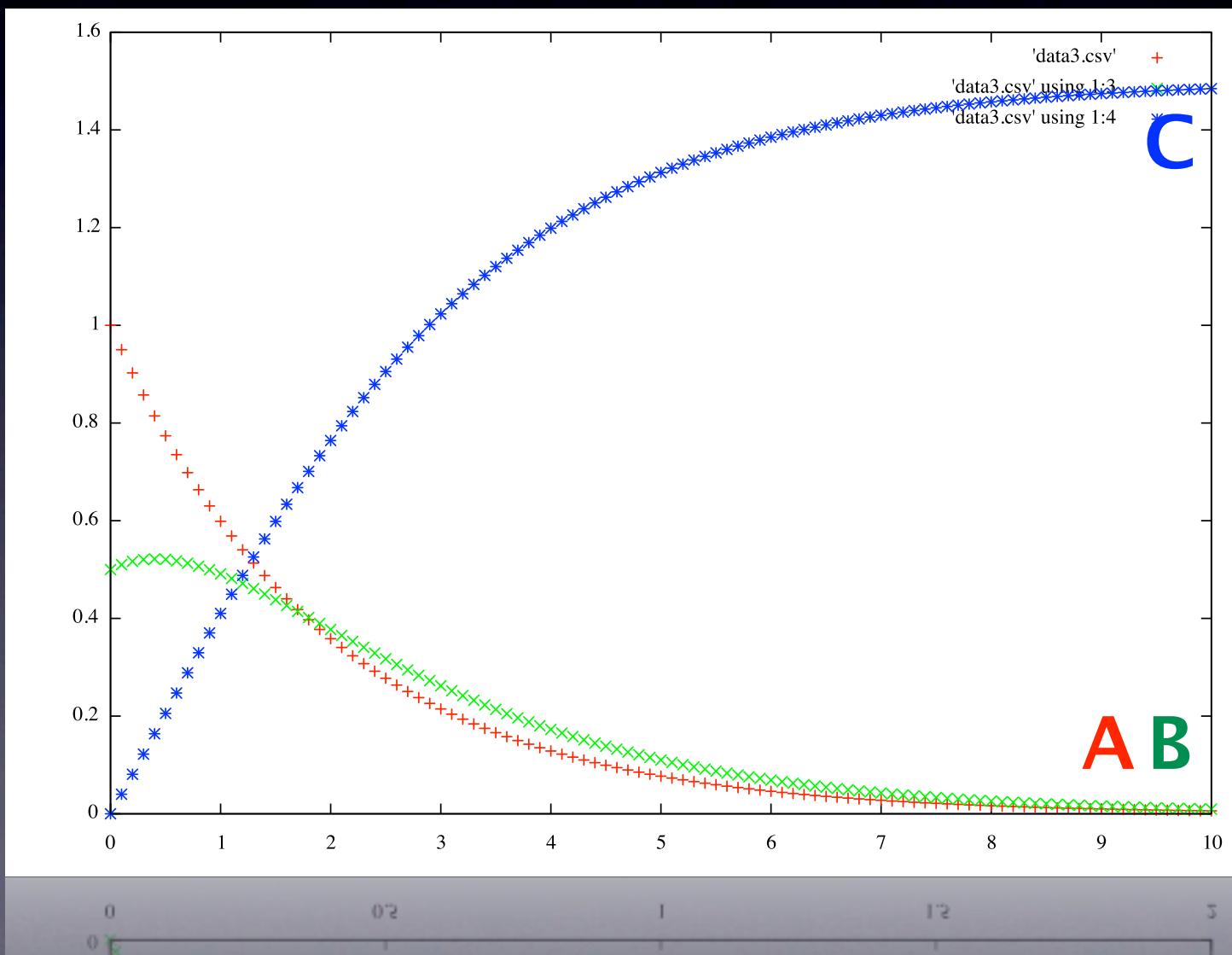
$$[A]_0 = 1.0, \quad [B]_0 = 0.5, \quad [C]_0 = 0.0$$

$$k_1 = 0.5, \quad k_2 = 0.8$$

3変数モデル

$$v_1 = k_1[A]$$

$$v_2 = k_2[B]$$



```
#include<stdio.h>

int main(void) {
    int i;
    double t = 0.0;
    double dt = 0.1;
    double v1, v2;
    double A = 1.0, B = 0.5, C = 0.0;
    double k1 = 0.5, k2 = 0.8;

    for (i = 0; i <= 100; i++) {
        printf("%lf, %lf, %lf, %lf\n", t, A, B, C);
        v1 = k1 * A;
        v2 = k2 * B;
        A = A - v1*dt; /* dA/dt = -v1 */
        B = B + v1*dt - v2*dt; /* dB/dt = v1 - v2 */
        C = C + v2*dt; /* dC/dt = v2 */
        t = t + dt;
    }
    return 0;
}
```

C言語版

```
function xdot = sim3(time,x)
```

```
xdot = zeros(3,1);
```

```
k1 = 0.5;
```

```
k2 = 0.8;
```

```
if (nargin == 0)
```

```
    xdot(1) = 1.0; % A
```

```
    xdot(2) = 0.5; % B
```

```
    xdot(3) = 0.0; % C
```

```
else
```

```
    A = x(1);
```

```
    B = x(2);
```

```
    C = x(3);
```

```
    v1 = k1 * A;
```

```
    v2 = k2 * B;
```

```
    xdot(1) = -v1; % dA/dt
```

```
    xdot(2) = v1 - v2; % dB/dt
```

```
    xdot(3) = v2; % dC/dt
```

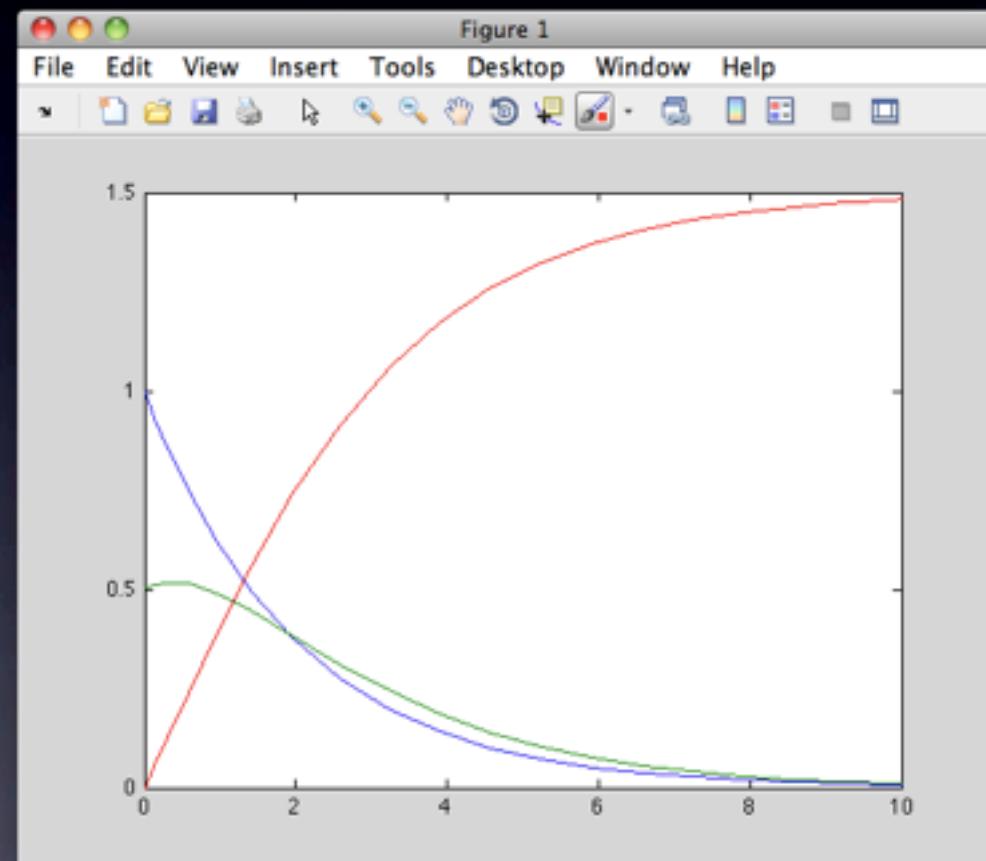
```
end;
```

変数の数

初期値

MATLAB版

sim3.m



```
>> [t,x] = ode23(@sim3,[0 10],sim3);  
>> plot(t,x)
```

Octave-Forge
odepkg でも可

本格的なモデル

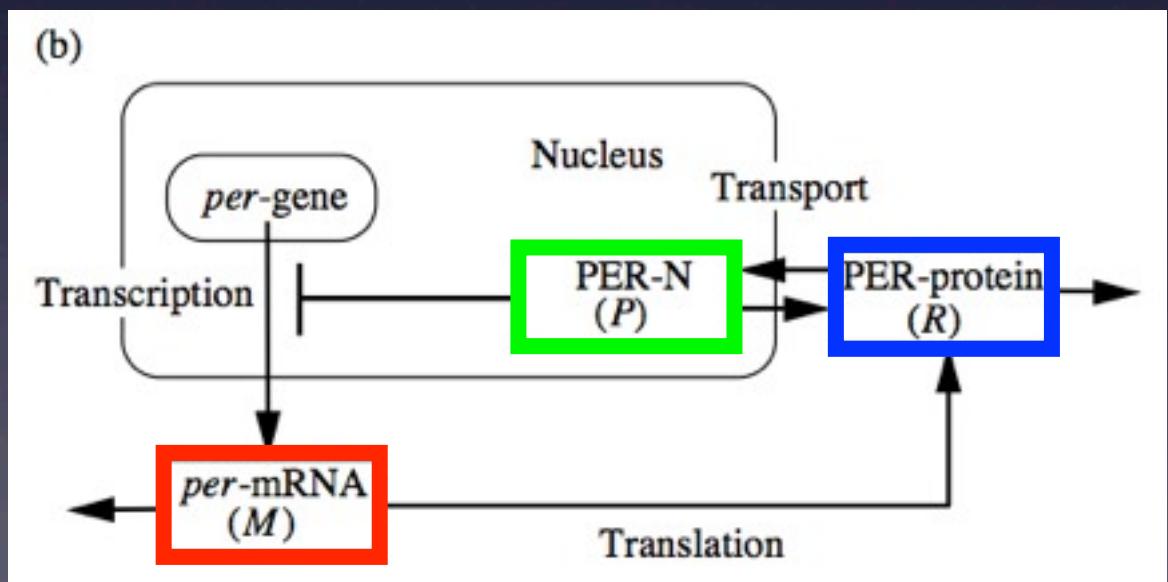
Circadian clock model

- mRNA(M)の生産は核内のタンパク質(P)によって抑制される
- Mは細胞質でタンパク質(R)に翻訳される
- P / R はそれぞれ細胞質 / 核内に輸送される

$$\frac{dM}{dt} = \frac{1}{1 + (P/h)^n} - aM,$$

$$\frac{dR}{dt} = sM - (d + u)R + vP,$$

$$\frac{dP}{dt} = uR - vP.$$



J. theor. Biol. (2002) 216, 193-208

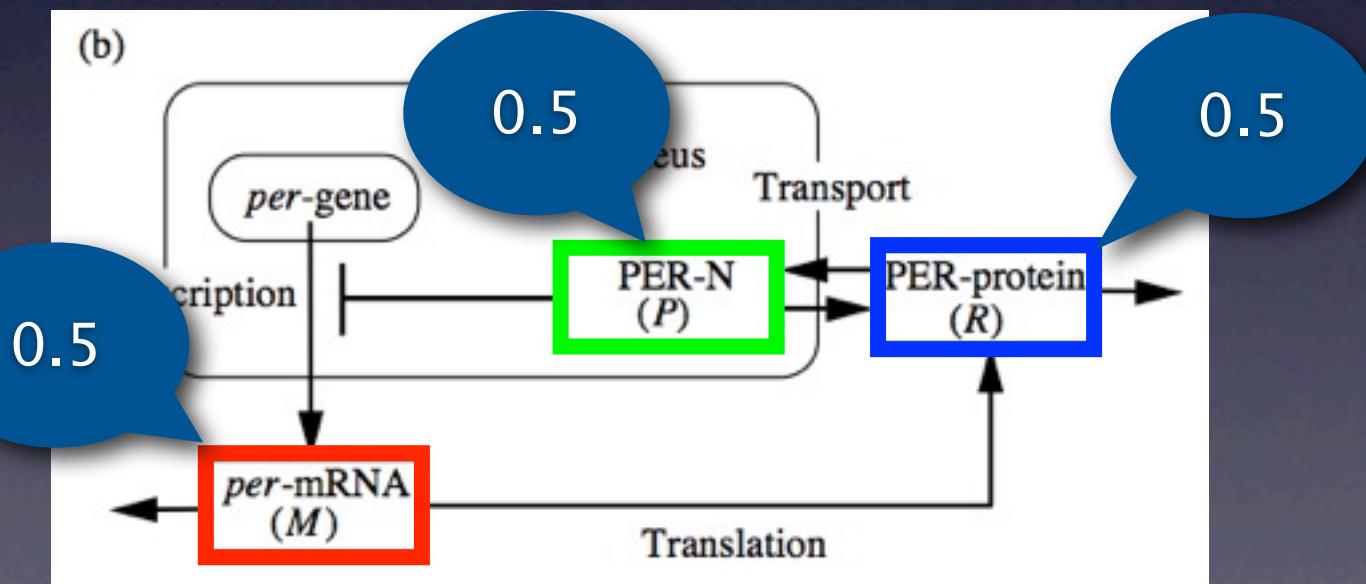
Circadian clock model

$$\frac{dM}{dt} = \frac{1}{1 + (P/h)^n} - aM,$$

$$\frac{dR}{dt} = sM - (d + u)R + vP,$$

$$\frac{dP}{dt} = uR - vP.$$

$$\begin{aligned} a &= s = d = v &= 1.0 \\ u &= 0.1 \\ h &= 0.01 \\ n &= 40 \end{aligned}$$



Circadian clock model

$$\frac{dM}{dt} = \frac{1}{1 + (P/h)^n} - aM,$$

$$\frac{dR}{dt} = sM - (d + u)R + vP,$$

$$\frac{dP}{dt} = uR - vP.$$

$$\begin{aligned} a &= s = d = v &= 1.0 \\ u &= 0.1 \\ h &= 0.01 \\ n &= 40 \end{aligned}$$

$1 / (1 + (P/h)^n)$

0.5

uR

0.5

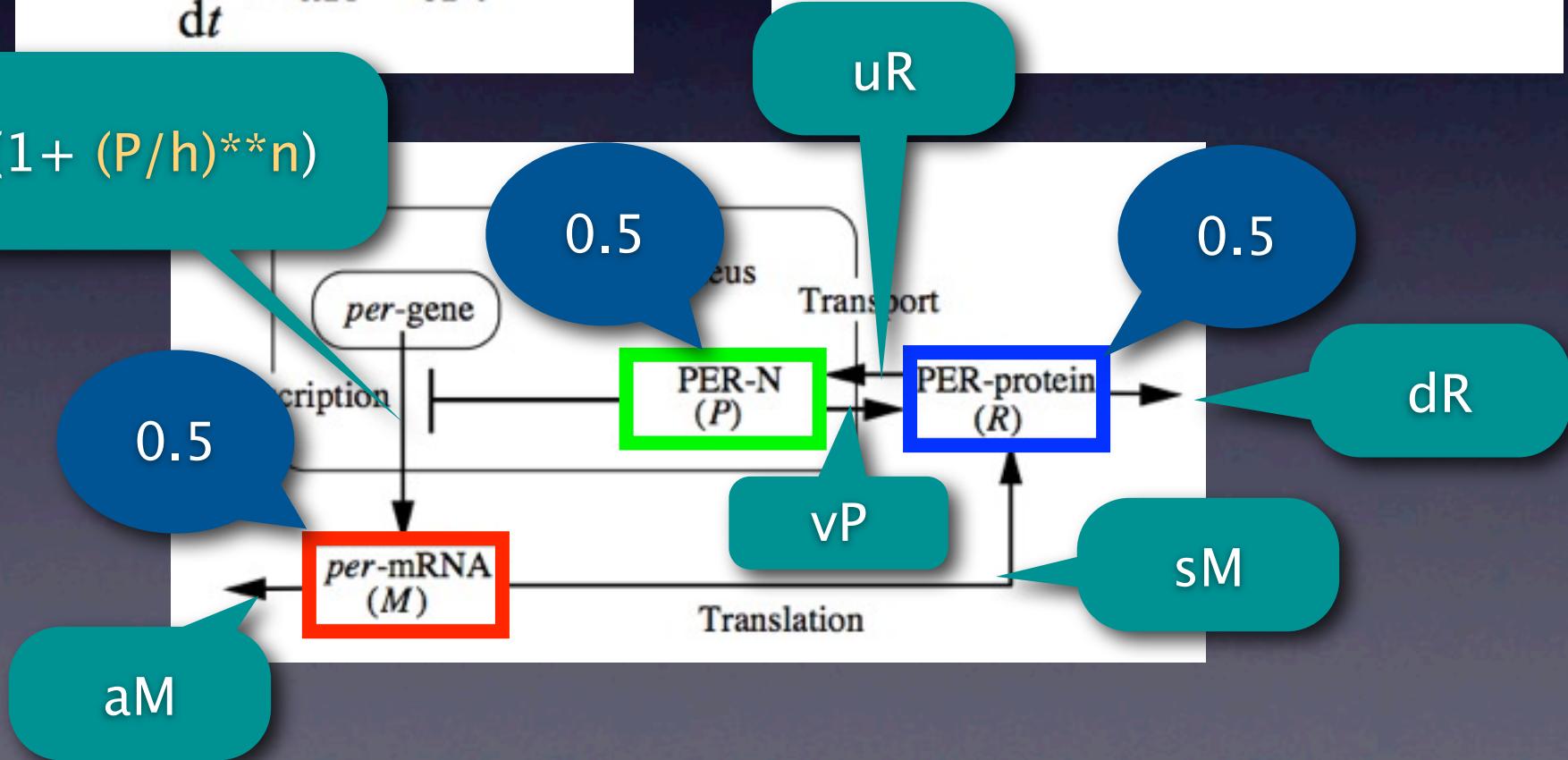
0.5

dR

vP

sM

aM



```

#!/usr/bin/perl

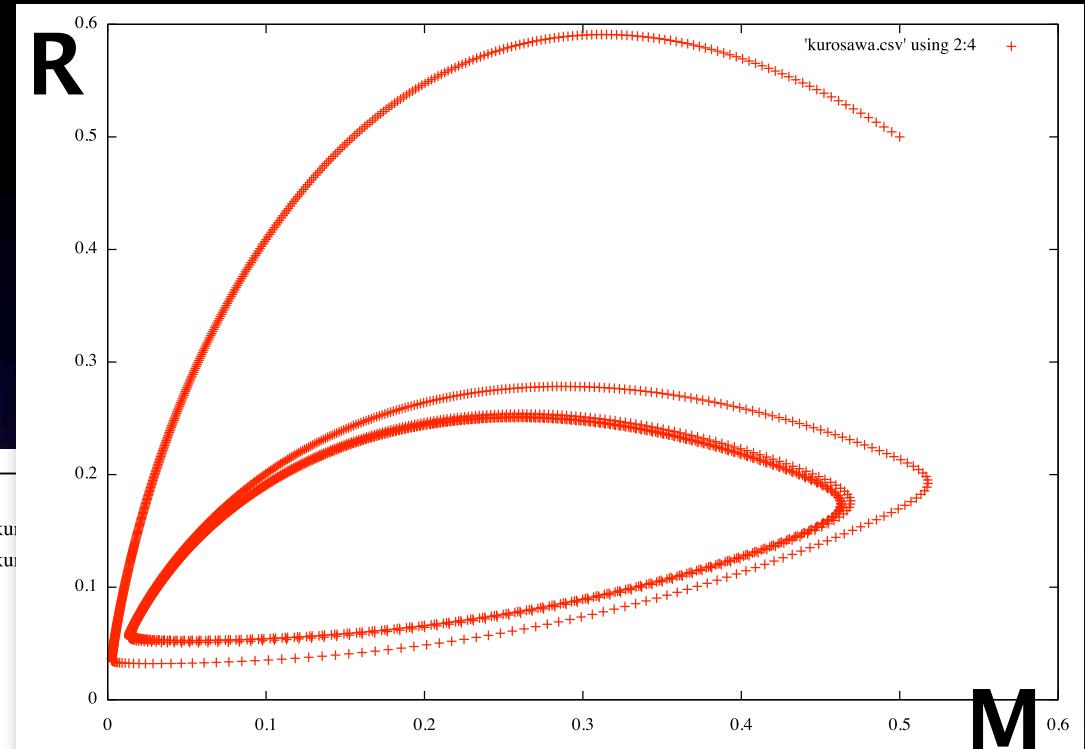
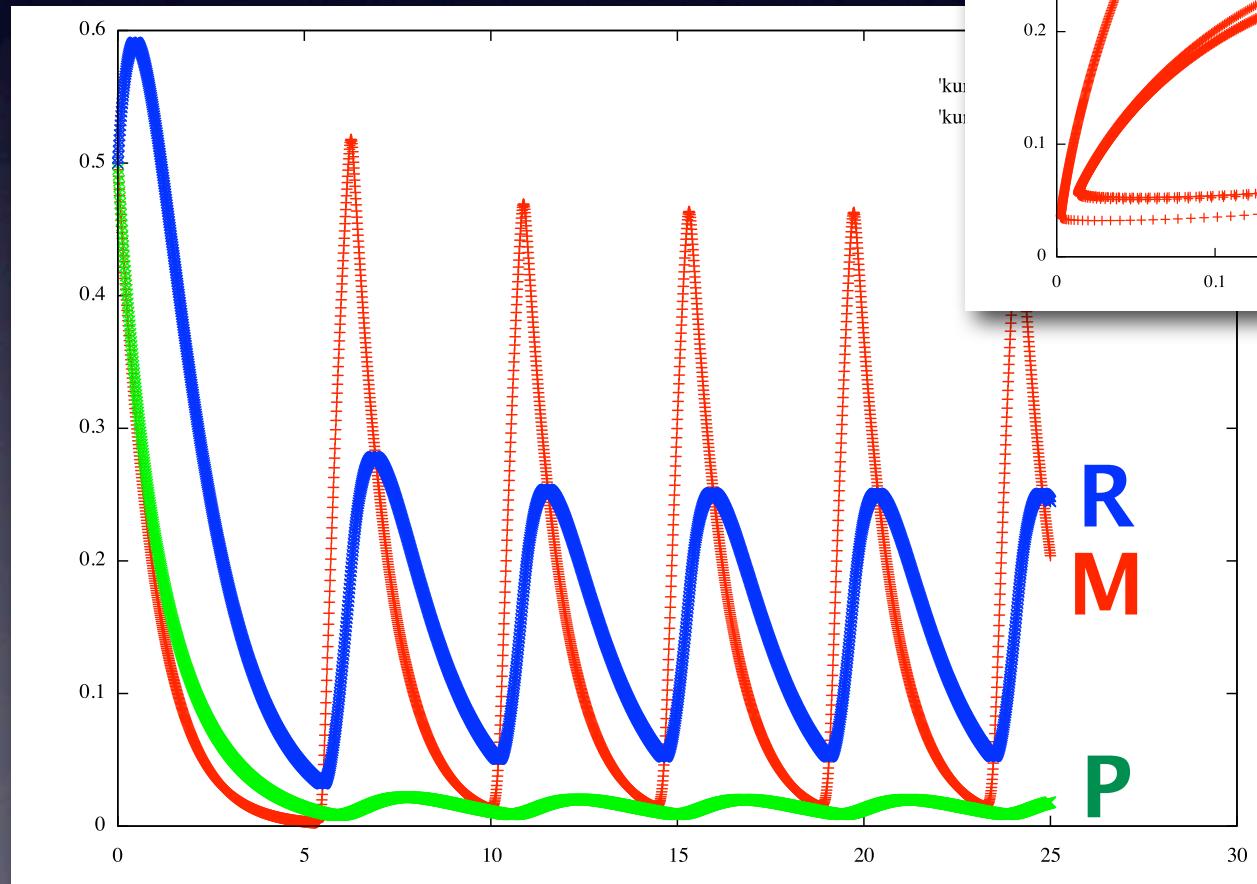
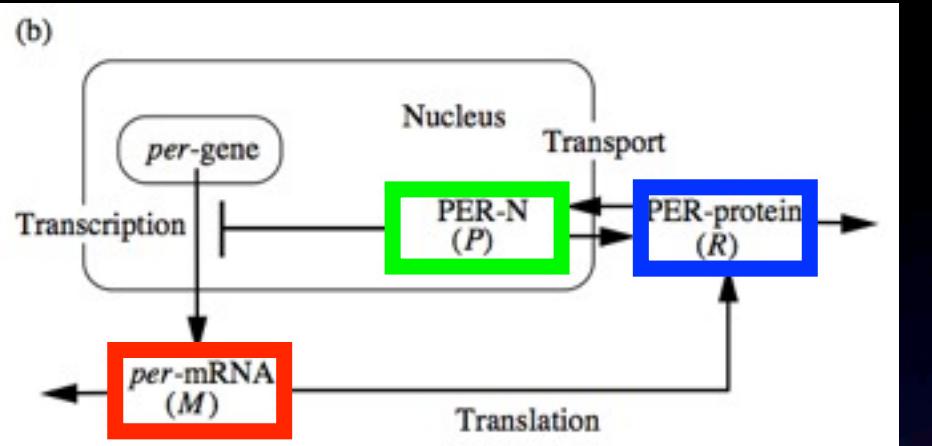
# Time variables
$t = 0.0;
$dt = 0.01;
# Parameters
$a = 1.0; $s = 1.0; $d = 1.0; $v = 1.0;
$u = 0.1; $h = 0.01;
$n = 40;
# Molecules
$M = 0.5; $P = 0.5; $R = 0.5;

for ($i = 0; $i <= 2500; $i++) {
    print"$t,$M,$P,$R\n";
    $dMdt = 1 / (1 + ($P/$h)**$n) - $a * $M;
    $dRdt = $s*$M - ($d+$u)*$R + $v*$P;
    $dPdt = $u*$R - $v*$P;
    $M = $M + $dMdt * $dt;
    $R = $R + $dRdt * $dt;
    $P = $P + $dPdt * $dt;
    $t = $t + $dt;
}

```

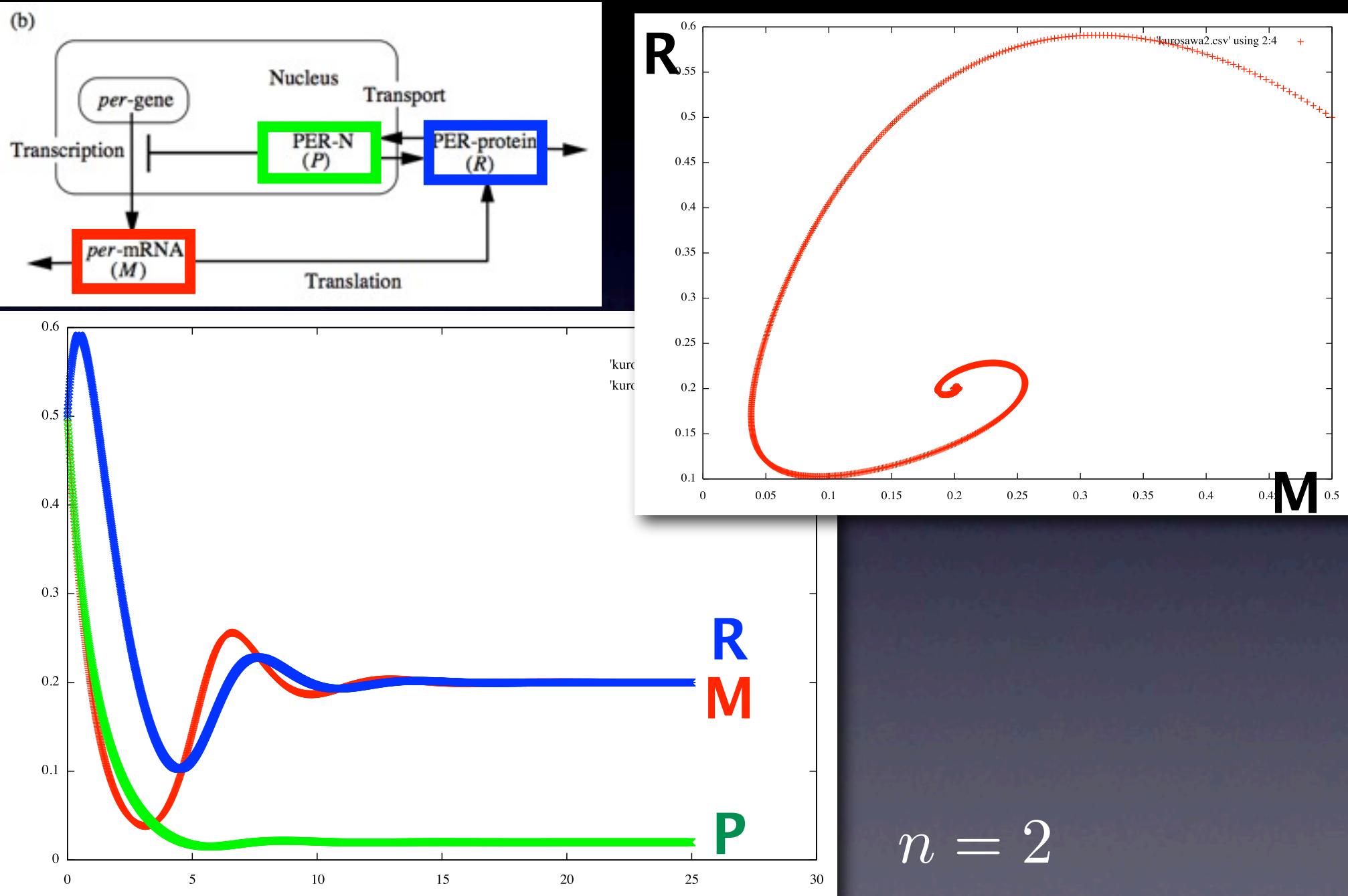
Perl版

Circadian clock model



$n = 40$

Circadian clock model



C言語版

```
#include<stdio.h>
#include<math.h>
int main(void) {
    int i;
    /* Time variables */
    double t = 0.0, dt = 0.01;
    /* Parameters */
    double a = 1.0, s = 1.0, d = 1.0, v = 1.0;
    double u = 0.1, h = 0.01;
    double n = 40;
    /* Molecules */
    double M = 0.5, P = 0.5, R = 0.5;
    double dMdt, dRdt, dPdt;

    for (i = 0; i <= 2500; i++) {
        printf("%lf, %lf, %lf, %lf\n", t, M, P, R);
        dMdt = 1 / (1 + pow(P/h,n)) - a * M;
        dRdt = s*M - (d+u)*R + v*P;
        dPdt = u*R - v*P;
        M = M + dMdt * dt;
        R = R + dRdt * dt;
        P = P + dPdt * dt;
        t = t + dt;
    }
    return 0;
}
```

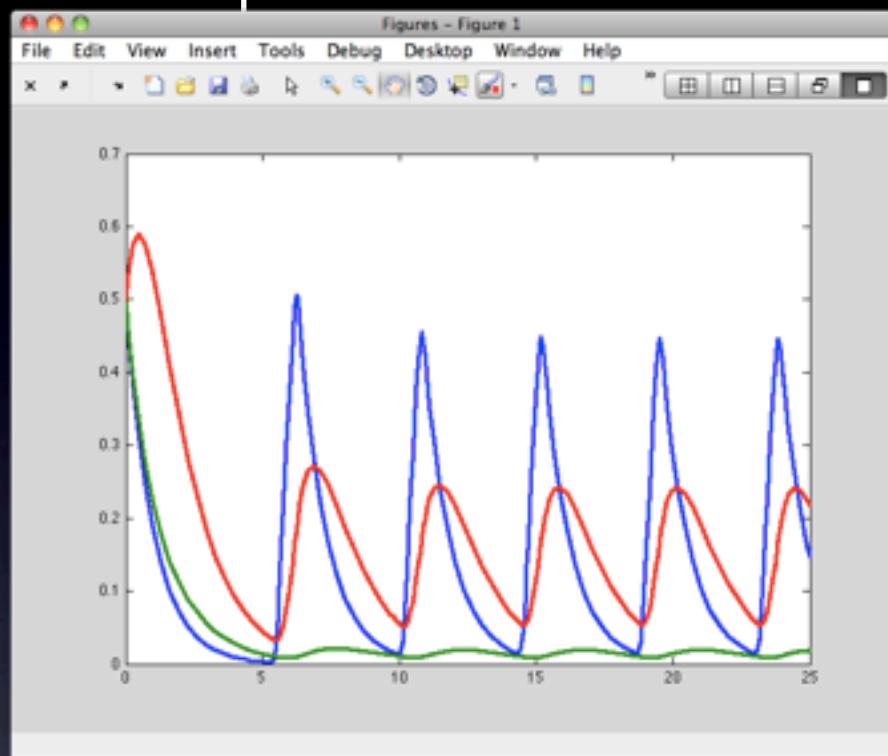
```

function xdot = kurosawa(time,x)
xdot = zeros(3,1);
a = 1.0; s = 1.0; d = 1.0; v = 1.0;
u = 0.1; h = 0.01;
n = 40;

if (nargin == 0)
    xdot(1) = 0.5; % dM
    xdot(2) = 0.5; % dP
    xdot(3) = 0.5; % dR
else
    M = x(1);
    P = x(2);
    R = x(3);
    xdot(1) = 1/(1+(P/h)^n) - a*M; % dM/dt
    xdot(2) = u*R - v*P; % dP/dt
    xdot(3) = s*M - (d+u)*R + v*P; % dR/dt
end;

```

MATLAB版



kurosawa.m

Octave-Forge
odepkg でも可

```

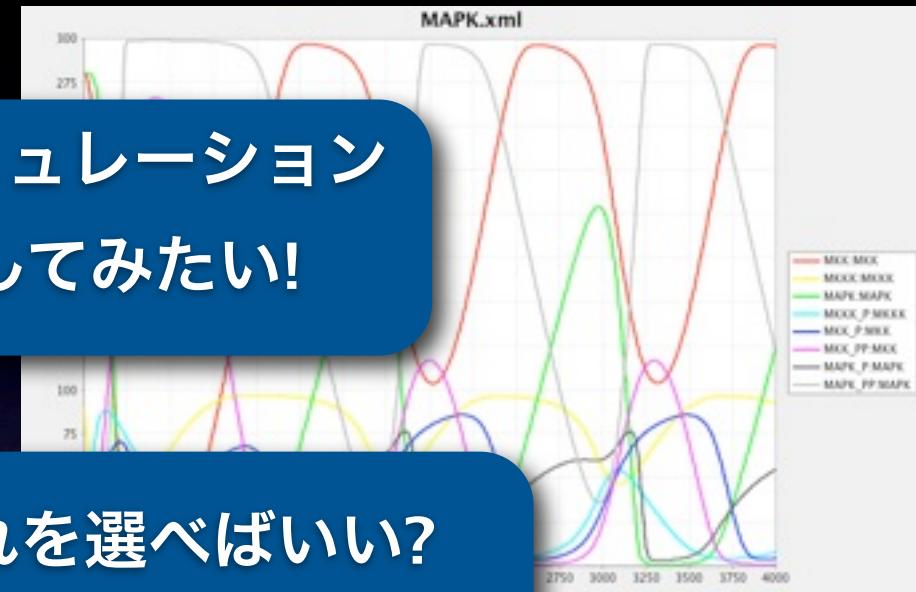
>> [t,x] = ode23(@kurosawa, [0 10], kurosawa);
>> plot(t,x)

```

今日の目標

- 数理モデルの構築
- シミュレータの実装
- シミュレーション

シミュレーション
してみたい!



どれを選べばいい?

Table 1. Kinetic equations comprising the computational model of the MAPK cascade.

$$d[MKKK]/dt = v_2 - v_1$$

$$d[MKKK-P]/dt = v_1 - v_2$$

$$d[MKK]/dt = v_6 - v_3$$

$$d[MKK-P]/dt = v_3 + v_5 - v_4 - v_6$$

$$d[MKK-PP]/dt = v_4 - v_5$$

$$d[MAPK]/dt = v_{10} - v_7$$

$$d[MAPK-P]/dt = v_7 + v_9 - v_8 - v_{10}$$

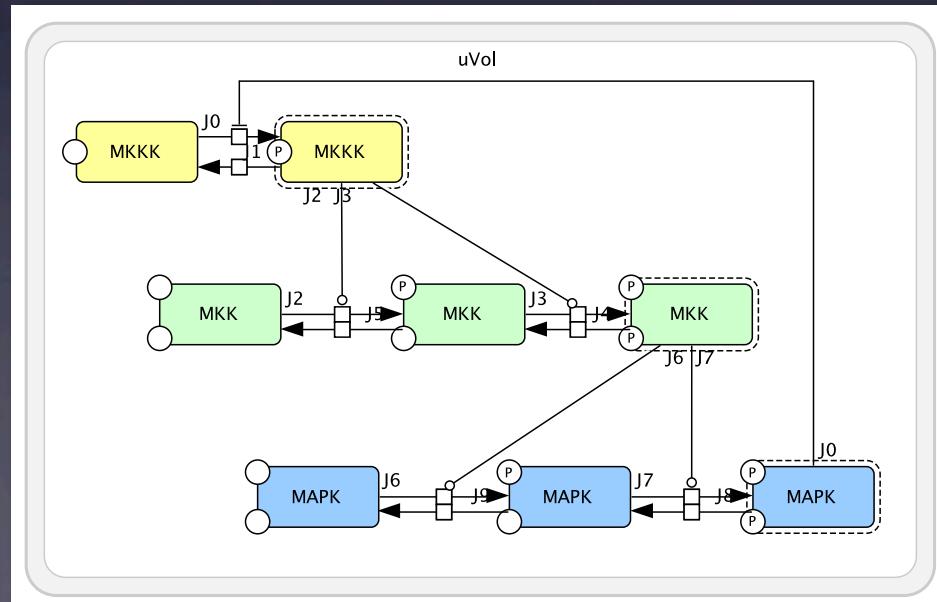
$$d[MAPK-PP]/dt = v_8 - v_9$$

Moiety conservation relations:

$$[MKKK]_{\text{total}} = [MKKK] + [MKKK-P]$$

$$[MKK]_{\text{total}} = [MKK] + [MKK-P] + [MKK-PP]$$

$$[MAPK]_{\text{total}} = [MAPK] + [MAPK-P] + [MAPK-PP]$$



ここから少し難しくなります

数値計算での注意点

● 精度

丸め誤差

- 浮動小数点数 (単精度と倍精度)

● 数値積分

打ち切り誤差

- 誤差とTaylor展開と刻み幅

- 硬い(stiff)微分方程式 (陽解法、陰解法)

● 乱数の周期性

● 速度

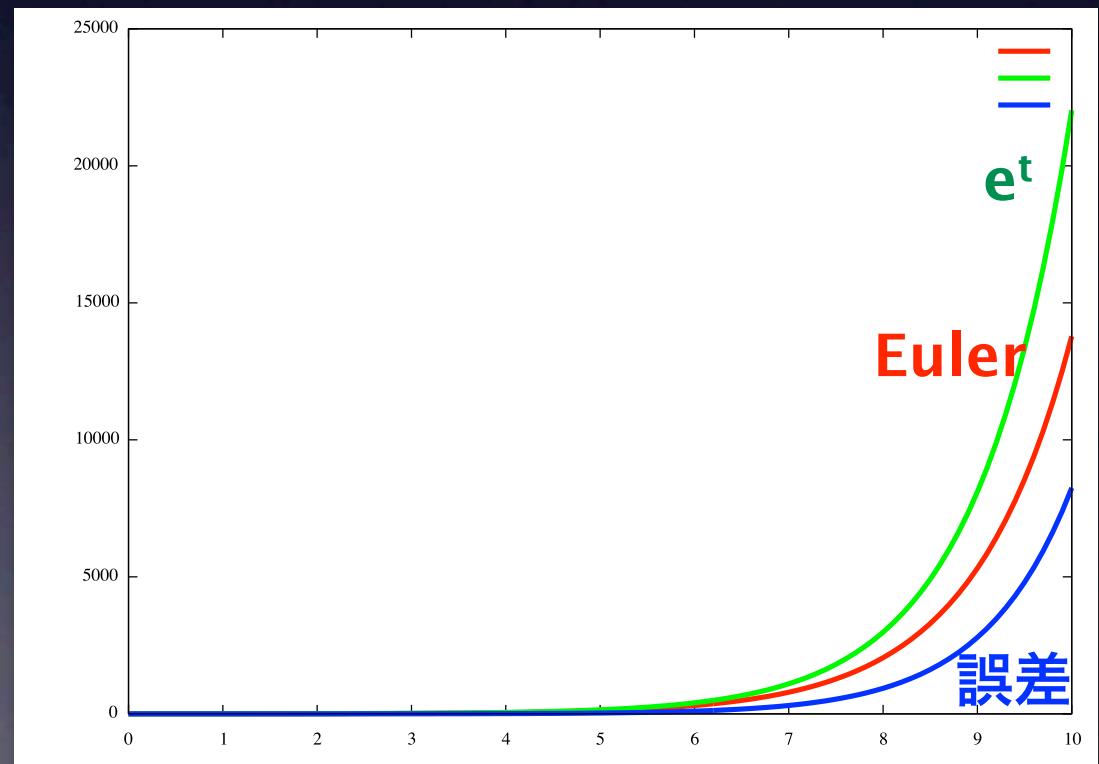
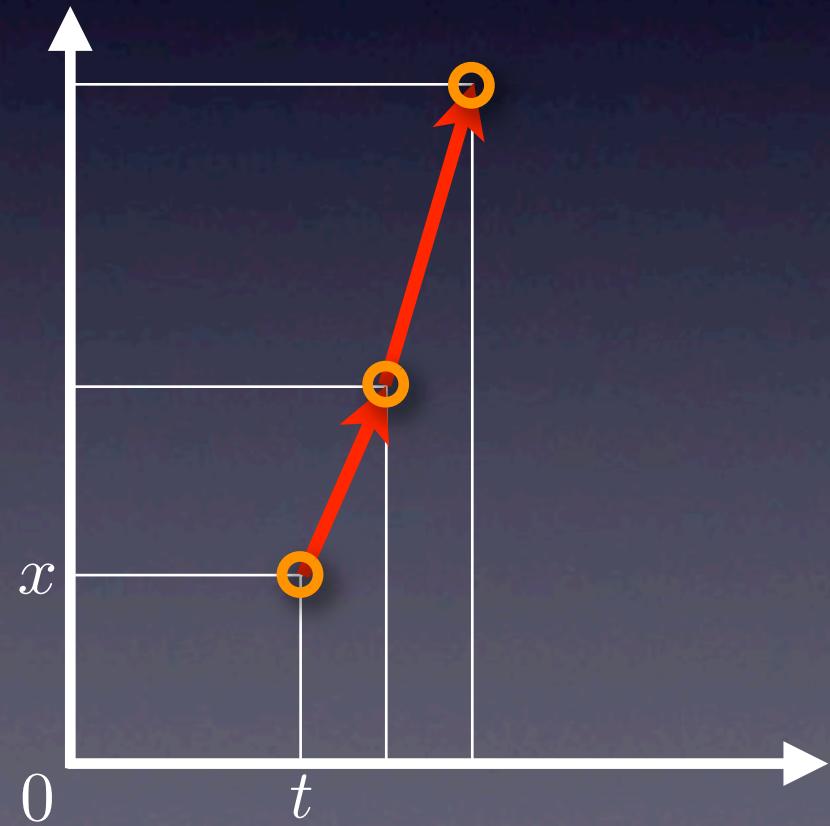
- 刻み幅(精度と計算時間のトレードオフ)

- 高速化(アルゴリズム、言語、ハードウェア)

Euler法と誤差

$$\frac{dx}{dt} = x \quad (t = 0, x = 1)$$

$$x = e^t$$

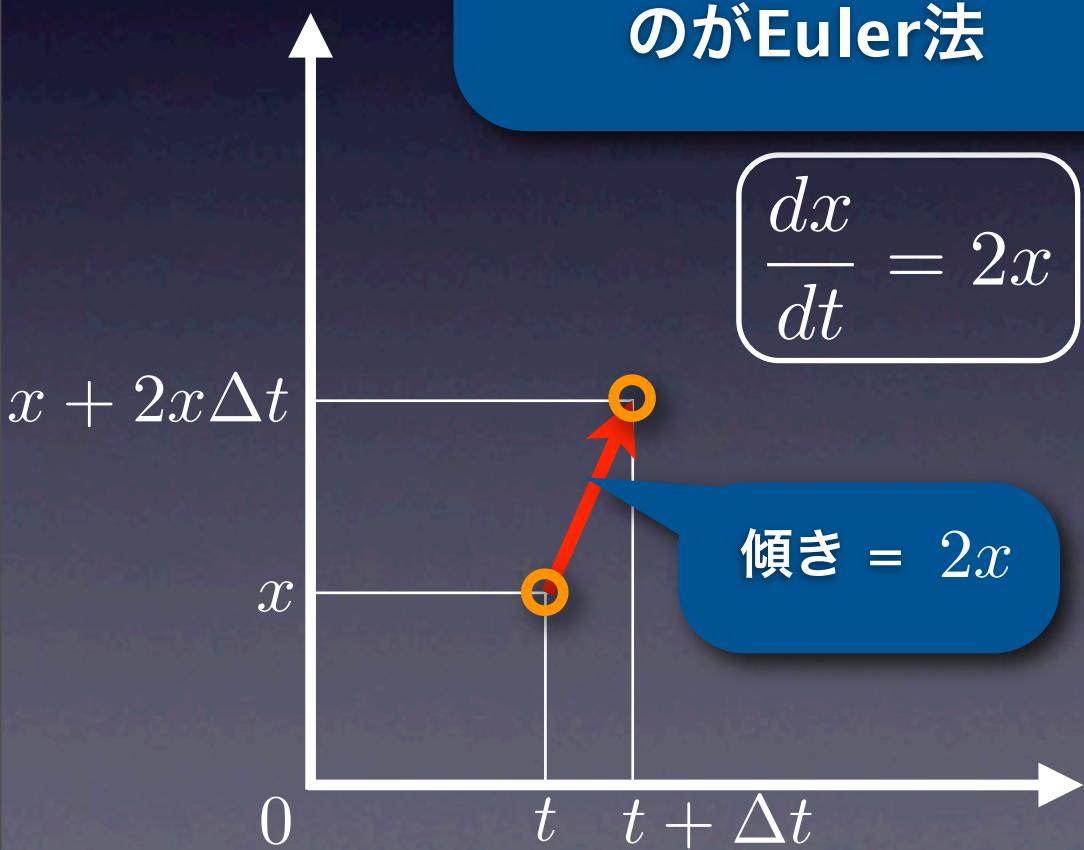


Taylor展開とEuler法

$$x(t + \Delta t) = x(t) + \Delta t x'(t) + \frac{1}{2} \Delta t^2 x''(t) + \frac{1}{3!} \Delta t^3 x^{(3)}(t) + \dots$$

ここまで打ち切った
のがEuler法

$$\dots + \frac{1}{n!} \Delta t^n x^{(n)}(t)$$



$$\text{打ち切り誤差} = O(\Delta t^2)$$

刻み幅を $1/2$ にすれば
誤差は $1/4$

Ruge-Kutta法(4次)

$$x(t + \Delta t) = x(t) + \Delta t x'(t) + \frac{1}{2} \Delta t^2 x''(t) + \frac{1}{3!} \Delta t^3 x^{(3)}(t) + \dots$$

$$\dots + \frac{1}{n!} \Delta t^n x^{(n)}(t)$$

$$d_1 = \Delta t \cdot f(t, x)$$

$$d_2 = \Delta t \cdot f\left(t + \frac{\Delta t}{2}, x(t) + \frac{d_1}{2}\right)$$

$$d_3 = \Delta t \cdot f\left(t + \frac{\Delta t}{2}, x(t) + \frac{d_2}{2}\right)$$

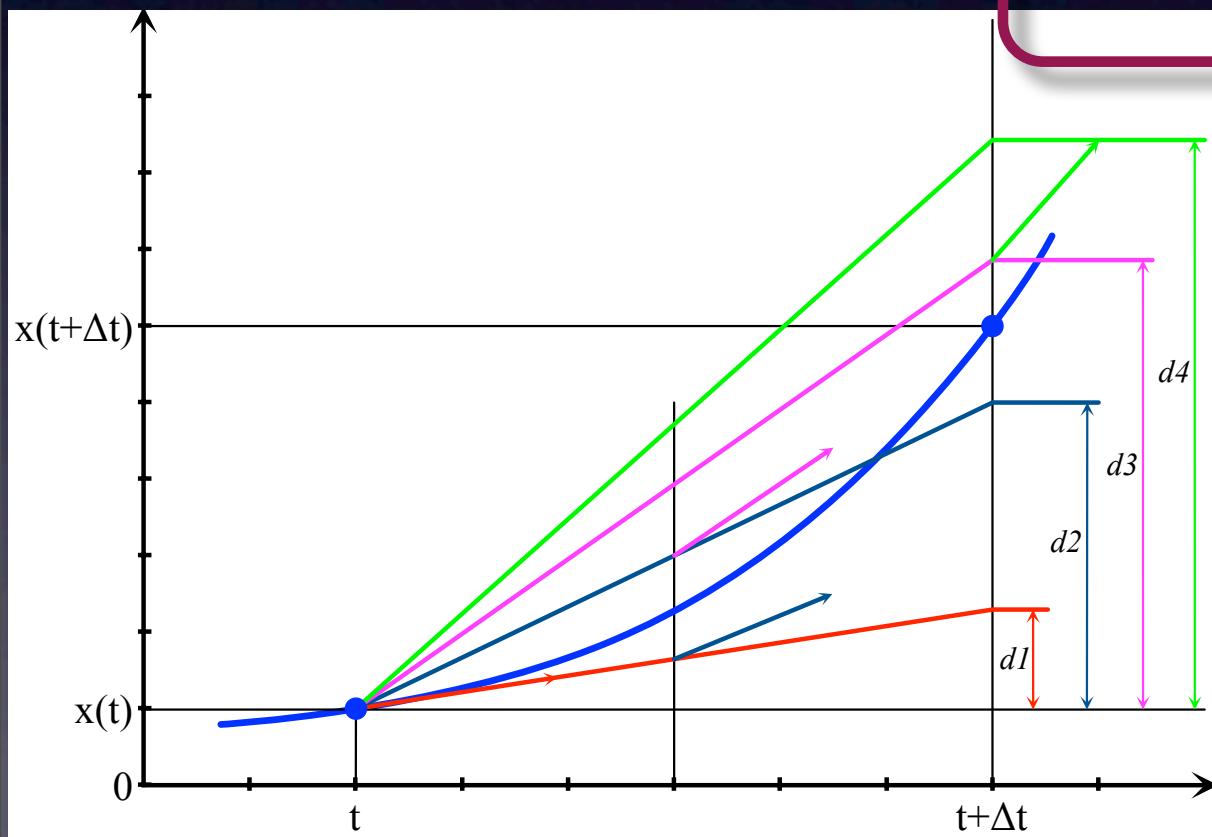
$$d_4 = \Delta t \cdot f(t + \Delta t, x(t) + d_3)$$

$$x(t + \Delta t) = x(t) + \frac{d_1 + 2d_2 + 2d_3 + d_4}{6}$$

打ち切り誤差

Ruge-Kutta法(4次)

$$x(t + \Delta t) = x(t) + \Delta t x'(t) + \frac{1}{2} \Delta t^2 x''(t) + \frac{1}{3!} \Delta t^3 x^{(3)}(t) + \dots + \frac{1}{n!} \Delta t^n x^{(n)}(t)$$



打ち切り誤差

$$\text{打ち切り誤差} = O(\Delta t^5)$$

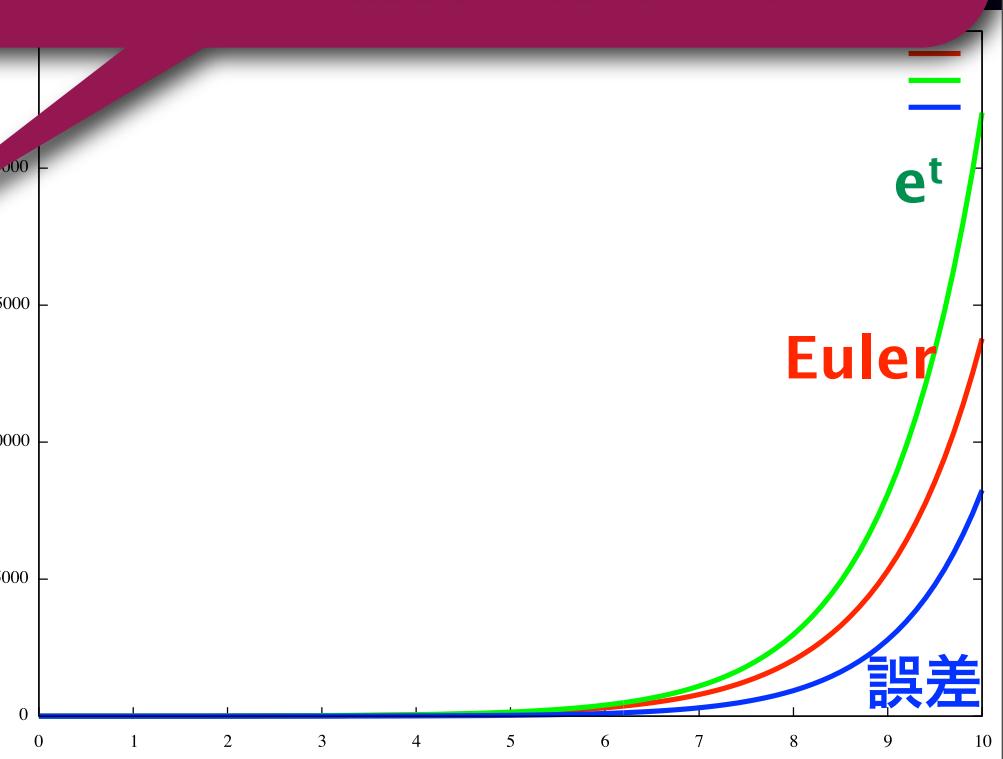
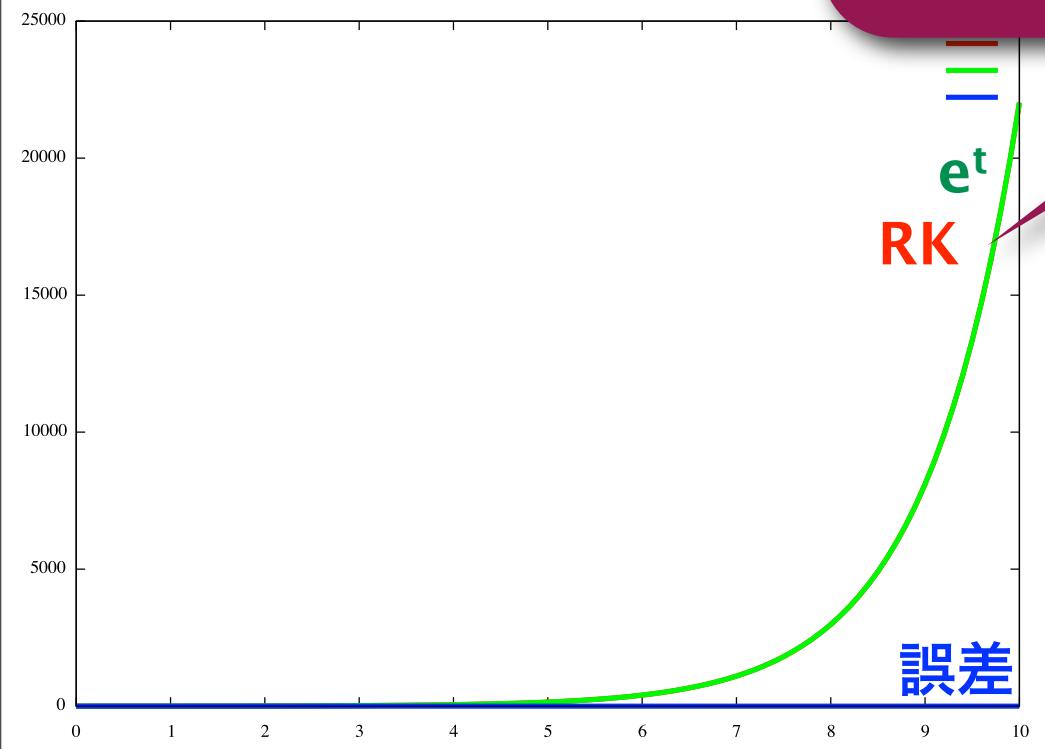
刻み幅を1/2にすれば
誤差は1/32

Ruge-Kutta法と誤差

$$\frac{dx}{dt} = x \quad (t = 0, x = 1)$$

$$x = e^t$$

計算時間は4倍になるが、打ち切り誤差を
小さくできるため精度は劇的に向上



Perl版

```
#!/usr/bin/perl

$dt = 0.1;
$t = 0.0;
$x = 1.0;
for ($i = 0; $i <= 100; $i++) {
    print "$t,$x\n";
    $d1 = &dxdt($t, $x) * $dt;
    $d2 = &dxdt($t+$dt/2, $x+$d1/2) * $dt;
    $d3 = &dxdt($t+$dt/2, $x+$d2/2) * $dt;
    $d4 = &dxdt($t+$dt, $x+$d3) * $dt;
    $dx = ($d1 + 2*$d2 + 2*$d3 + $d4) / 6;
    $x = $x + $dx;
    $t = $t + $dt;
}
```

```
sub dxdt {
    my ($t, $x) = @_;
    my $dxdt = $x;      # dx/dt = x
    return $dxdt;
}
```

C言語版

```
#include<stdio.h>

double dxdt(double t, double x) {
    double dxdt = x; /* dx/dt = x */
    return dxdt;
}
```

```
int main(void) {
    int i;
    double t = 0.0, dt = 0.1;
    double x = 1.0;
    double dx, d1, d2, d3, d4;

    for (i = 0; i <= 100; i++) {
        printf("%lf,%lf\n", t, x);
        d1 = dxdt(t, x) * dt;
        d2 = dxdt(t+dt/2, x+d1/2) * dt;
        d3 = dxdt(t+dt/2, x+d2/2) * dt;
        d4 = dxdt(t+dt, x+d3) * dt;
        dx = (d1 + 2*d2 + 2*d3 + d4) / 6;
        x = x + dx;
        t = t + dt;
    }
    return 0;
}
```

陽解法と陰解法

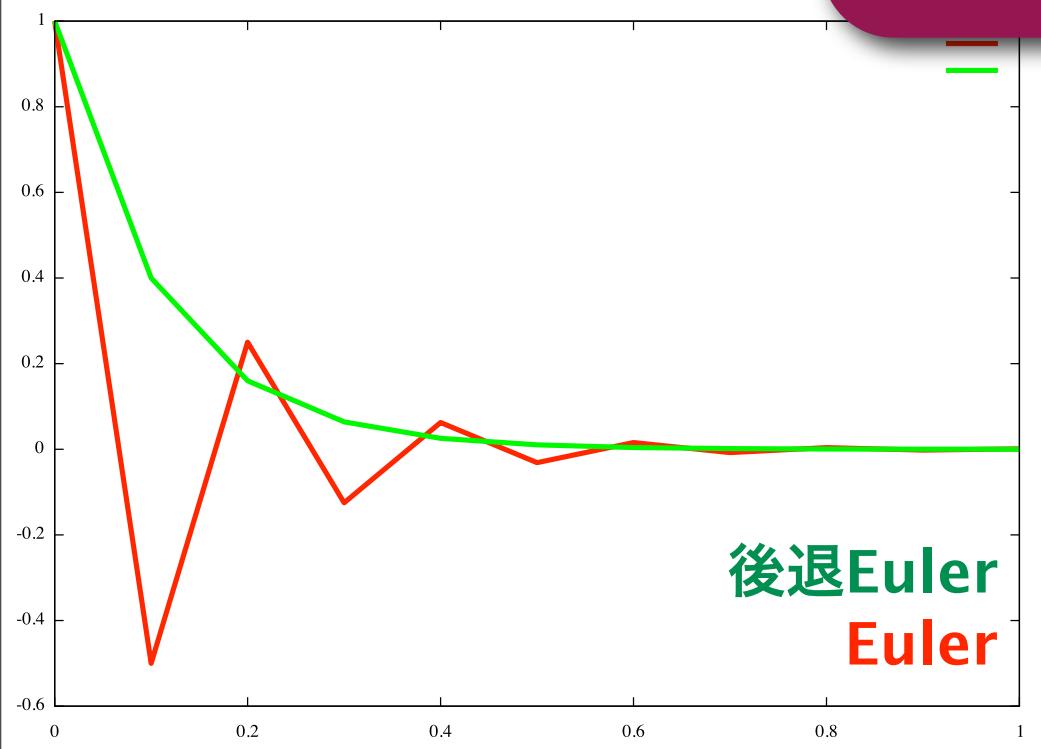
- 陽解法 (時刻 t までの x の値で $x(t + \Delta t)$ が求まる)
 - Euler法, Runge-Kutta法, Adams-Bashforth法
 - 硬い微分方程式が解けない(発散する)
 - 実装が容易
- dx/dtが非常に大きく変動するモデル、
dx/dt, dy/dtのオーダーが大きく異なるモデル
- 陰解法 ($x(t + \Delta t)$ を求めるのに $x(t + \Delta t)$ が必要)
 - 後退Euler法, Adams-Moulton法, Gear法
 - 硬い微分方程式に対応
 - 実装が面倒 (例: 数値微分, LU分解, ニュートン法, etc.)

Euler法と後退Euler法

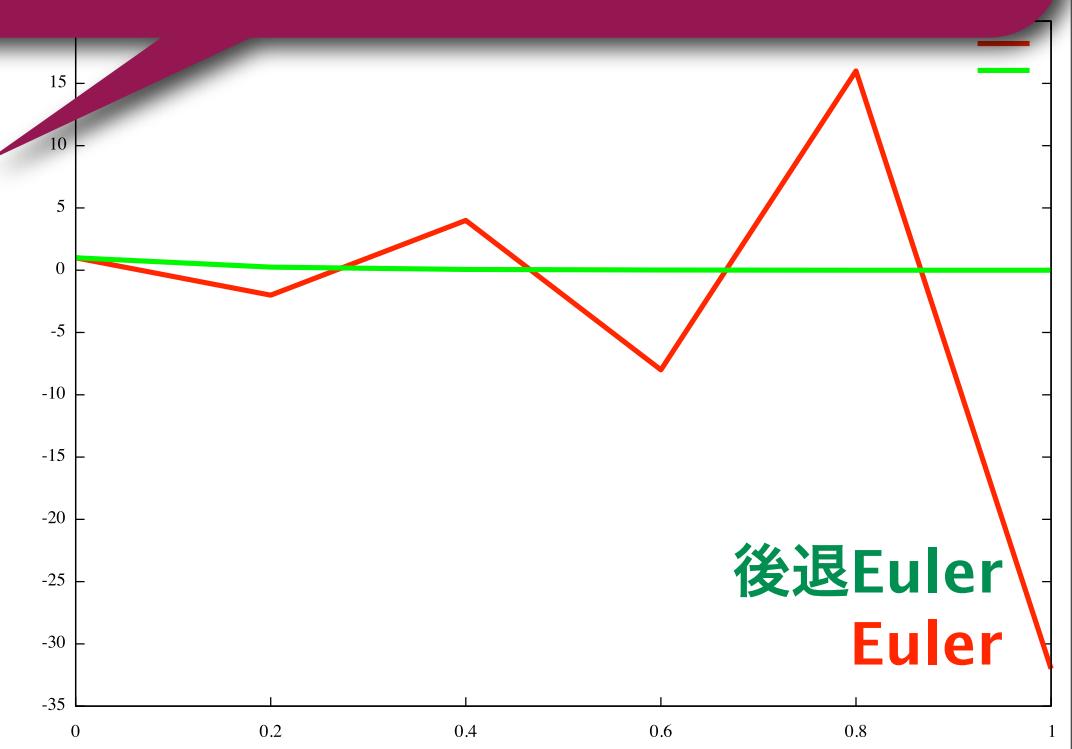
$$\frac{dx}{dt} = -15x \quad (t = 0, x = 1)$$

$$x = e^{-15t}$$

刻み幅によっては不安定になる、発散する



$$\Delta t = 0.1$$



$$\Delta t = 0.2$$

数値計算での注意点

● 精度

丸め誤差

- 浮動小数点数 (単精度と倍精度)

● 数値積分

打ち切り誤差

- 誤差とTaylor展開と刻み幅

- 硬い(stiff)微分方程式 (陽解法、陰解法)

● 乱数の周期性

● 速度

- 刻み幅(精度と計算時間のトレードオフ)

- 高速化(アルゴリズム、言語、ハードウェア)

浮動小数点数

コンピュータの中で数字はどう表現される?



2進法

2^3

2^2

2^1

2^0

1	0	1	1
---	---	---	---

$$\bullet 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1 = 11$$

実数はどう表現される？

```
$pi = 3.14;  
$sqr = 1.4142;
```

```
float pi = 3.14;  
double sqr = 1.4142;
```

まずは実験

```
#include<stdio.h>

int main(void) {
    int i;
    float sum = 0;
    float f = 1.1;

    for (i = 0; i < 10; i++) {
        sum = sum + f;
    }
    printf("sum = %f\n", sum);
    return 0;
}
```

1.1を10回足すだけ
のプログラム

?!
./a.out
sum = 11.000001

浮動小数点表記

- 10進数

- $123400000 = 1.234 \times 10^8$

- $0.001234 = 1.234 \times 10^{-3}$

- 2進数

- $11011000000 = 1.1011 \times 2^{10}$

- $0.0000010101 = 1.0101 \times 2^{-6}$

仮数

指数

float型

- 2進数

仮数

指数

- $11011000000 = 1.1011 \times 2^{10}$

- $0.0000010101 = 1.0101 \times 2^{-6}$

符号部 (1bit)

float型 = 32bit

0 0 1 1 1 1 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

指数部 (8bit)

仮数部 (23bit)

$$\text{floatの値} = (-1)^{\text{符号部}} \times 2^{\text{指数部} - 127} \times 1.\text{仮数部}$$

仮数部の計算方法

仮数

指数

$$0.0000010101 = 1.0101 \times 2^{-6}$$

$$2^{-1} \quad 2^{-2} \quad 2^{-3} \quad 2^{-4} \quad 2^{-5} \quad \dots \quad 2^{-22} \quad 2^{-23}$$

	0	1	0	1	0	...	0	0
--	---	---	---	---	---	-----	---	---

$$\begin{aligned}2^{-2} + 2^{-4} &= 0.25 + 0.0625 \\&= 0.3125\end{aligned}$$

double型

● 2進数

- $110110000000 = 1.1011 \times 2^{10}$
- $0.0000010101 = 1.0101 \times 2^{-6}$

符号部 (1bit)

double型 = 64bit



指数部 (11bit)

仮数部 (52bit)

0.1を2進数で表すと

$0.1_{(10)} = 0.0\text{0011}0011\text{0011}0011\dots_{(2)}$

```
#include<stdio.h>
```

```
int main(void) {
    int i;
    float sum = 0;
    float f = 1.1;

    for (i = 0; i < 10; i++) {
        sum += f;
    }
    printf("sum = %f\n", sum);
    return 0;
}
```

10進数の 0.1 は
2進数だと循環小数

丸め誤差

```
./a.out
sum = 11.000001
```

倍精度(double)で再挑戦

$0.1_{(10)} = 0.00011001100110011\dots_{(2)}$

```
#include<stdio.h>
```

```
int main(void) {
    int i;
    double sum = 0;
    double f = 1.1;

    for (i = 0; i < 10; i++) {
        sum += f;
    }
    printf("sum = %lf\n", sum);
    return 0;
}
```

10進数の 0.1 は
2進数だと循環小数

OK

```
./a.out
sum = 11.000000
```

浮動小数点数: まとめ

$$0.1_{(10)} = 0.0\boxed{00011}\boxed{0011}\boxed{0011}\boxed{0011}\dots_{(2)}$$

```
float f = 1.1;  
double f = 1.1;
```

- 単精度(float)は精度が悪い(有効数字6 or 7桁)
- 0.1より0.125の方が精度が高い($1/2^n$ は高精度)
- C言語を使う場合、floatは避ける
- floatとdoubleの計算時間の差は(今は)ない
- Perl等のスクリプト型言語は大概倍精度
- シミュレータが单精度か倍精度か確認すべき

余談: Javaのdouble

```
volatile double x, y, z, d;  
public void doTest() {  
    x = 9007199254740994.0; /* 2^53 + 2 */  
    y = 1.0 - 1/65536.0;  
    z = x + y;  
    d = z - x;  
    System.out.println("z = " + z);  
    System.out.println("d = " + d);  
}
```

z = 9.007199254740994E15
d = 0.0

gij 4.3.3 on
Linux/x86

z = 9.007199254740996E15
d = 2.0

OK

NG

数値計算での注意点: 結論

● 精度

- 単精度はだめ (C言語なら float ×, double ○)
- Euler法で満足してはだめ Runge-Kutta法等を実装
- 陰解法もがんばって実装するとgood
- C言語標準の rand() はかなりいまいち
MT(Mersenne Twister)等の長周期性、均等性をもつ
高速な擬似乱数生成器を利用すべき
- SUNDIALS, LSODA, GSL などのライブラリを利用

● 速度

- 高速化の前に精度の保証を!
- 全部自前で実装しないと大して速くならない

シミュレータの選び方

1. 数理モデル構築
2. シミュレーション
3. プロット
4. 解析(あれば)

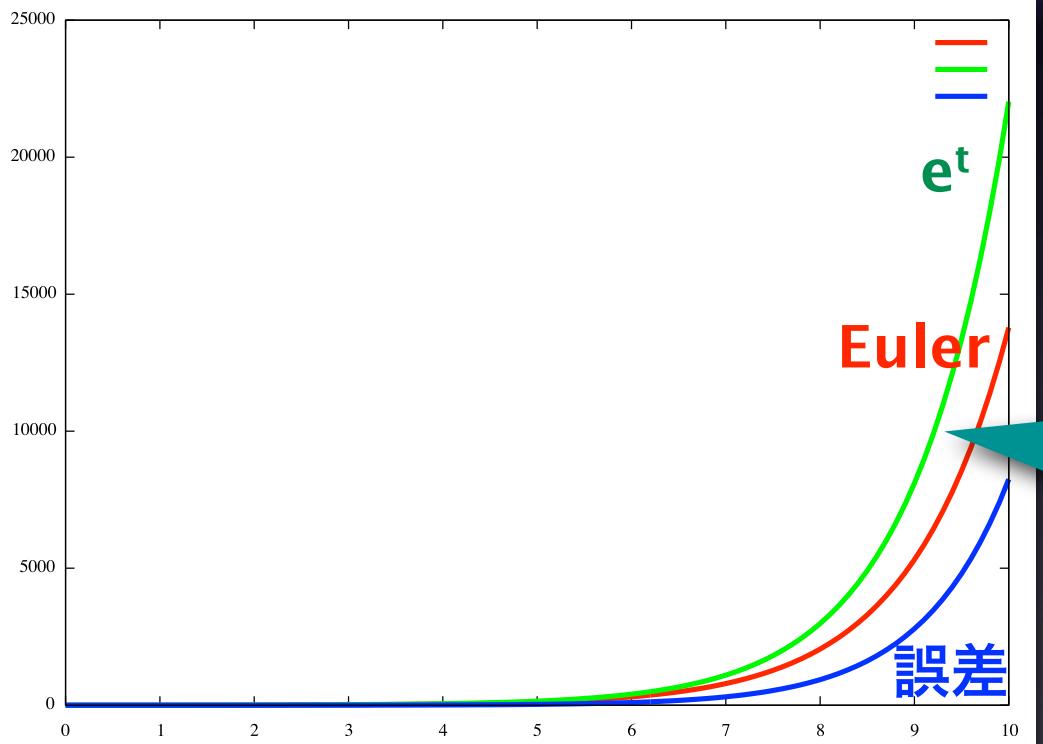
ポイントは

1. 精度
2. 使いやすさ
3. モデルを再利用可能か

精度

$$\frac{dx}{dt} = x \quad (t = 0, x = 1) \quad \Delta t = 0.1$$

$$x = e^t$$



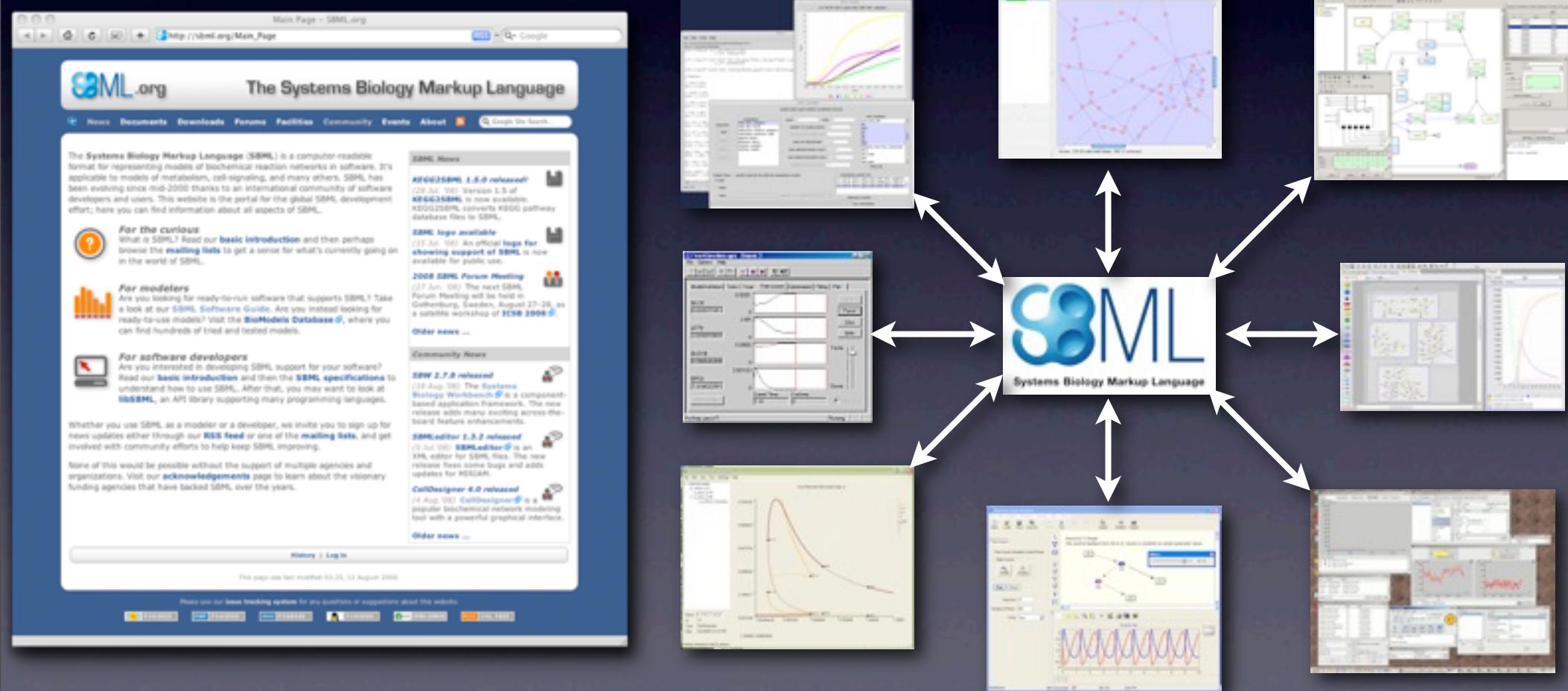
この条件でシミュレーションを実行、 $x = e^t$ と比較

- * SUNDIALS (CVODE)
- * LSODA
- * GSL 等

使用している数値計算
ライブラリが明記されて
いれば大丈夫

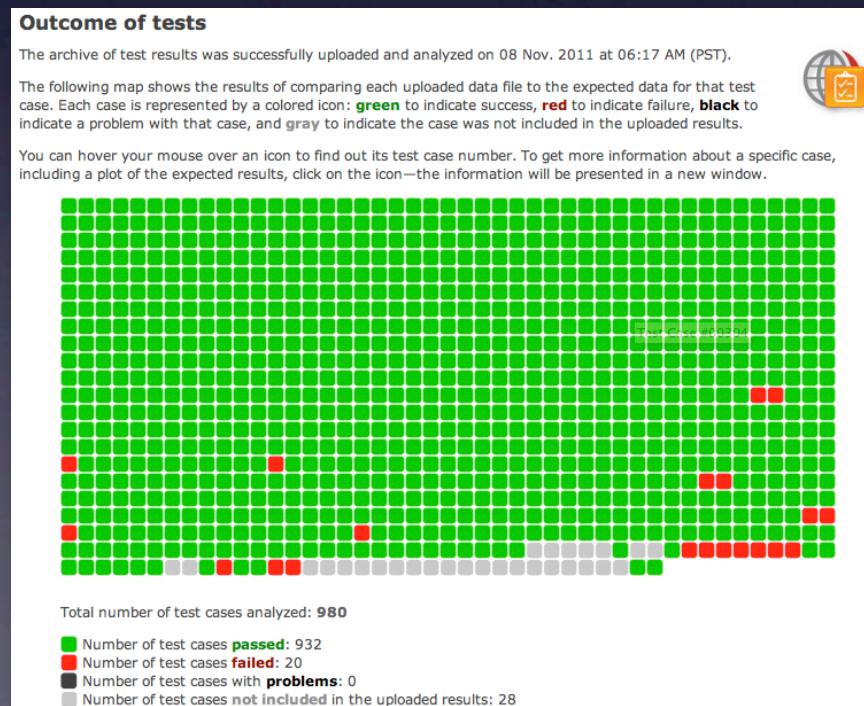
モデルの再利用

- 230以上のSBML対応ソフトウェア
- <http://sbml.org>



機能

- 常微分方程式の初期値問題を解くだけじゃない
- 代数微分方程式、イベント、遅延微分方程式、etc.
- SBMLのすべての仕様を満たす(テストをパスした)
シミュレータは存在しない



SBML.org The Systems Biology Markup Language

News Documents Downloads Forums Facilities Community Events About

Parent page: SBML.org / Facilities

Online SBML Test Suite

The SBML Test Suite allows you to evaluate the degree and correctness of SBML support implemented in SBML-compatible software. The system supports specifications of SBML up through **Level 3 Version 1 Core**. This page is the interface to the online version of the Suite; it allows you to upload test results and have them evaluated by our server.

There are three steps to using the online interface:

1. **Select and download test cases.** You can download all cases or select a subset using the interface provided on our online test case selection page.
2. **Run simulations** of the models in the software package you are testing, and collect the results. How you run the cases is up to you and the software you are testing.
3. **Upload the simulation results.** This online service will compare them to the expected results and provide you with a report of the outcome.

Limitations: (1) This online service works only with the semantic portion of the SBML Test Suite. It does not evaluate the results of syntactic tests available with the full SBML Test Suite. (2) Only HTTP is supported as the protocol for uploading results; other protocols such as https://, ftp://, etc. are not supported.

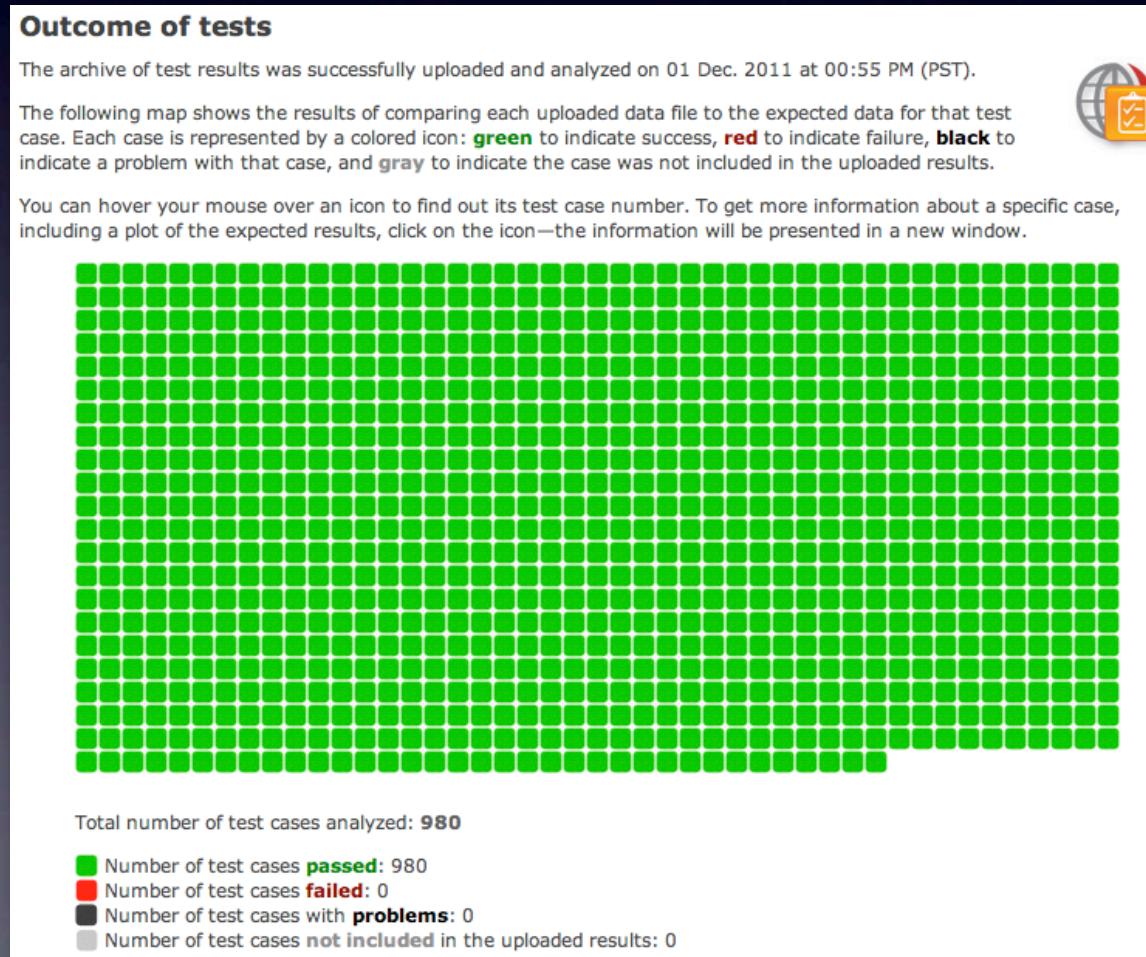
This Online SBML Test Suite server-based software was written by Michael Hucka and Kimberly Begley. The test cases were developed by Sarah H. Keating, Lucian Smith, and Michael Hucka. Please visit the main page of the **SBML Test Suite** for more information and other resources.

Please use our issue tracking system for any questions or suggestions about this website. This page was last modified 16-29, 6 June 2011.

newシミュレータ

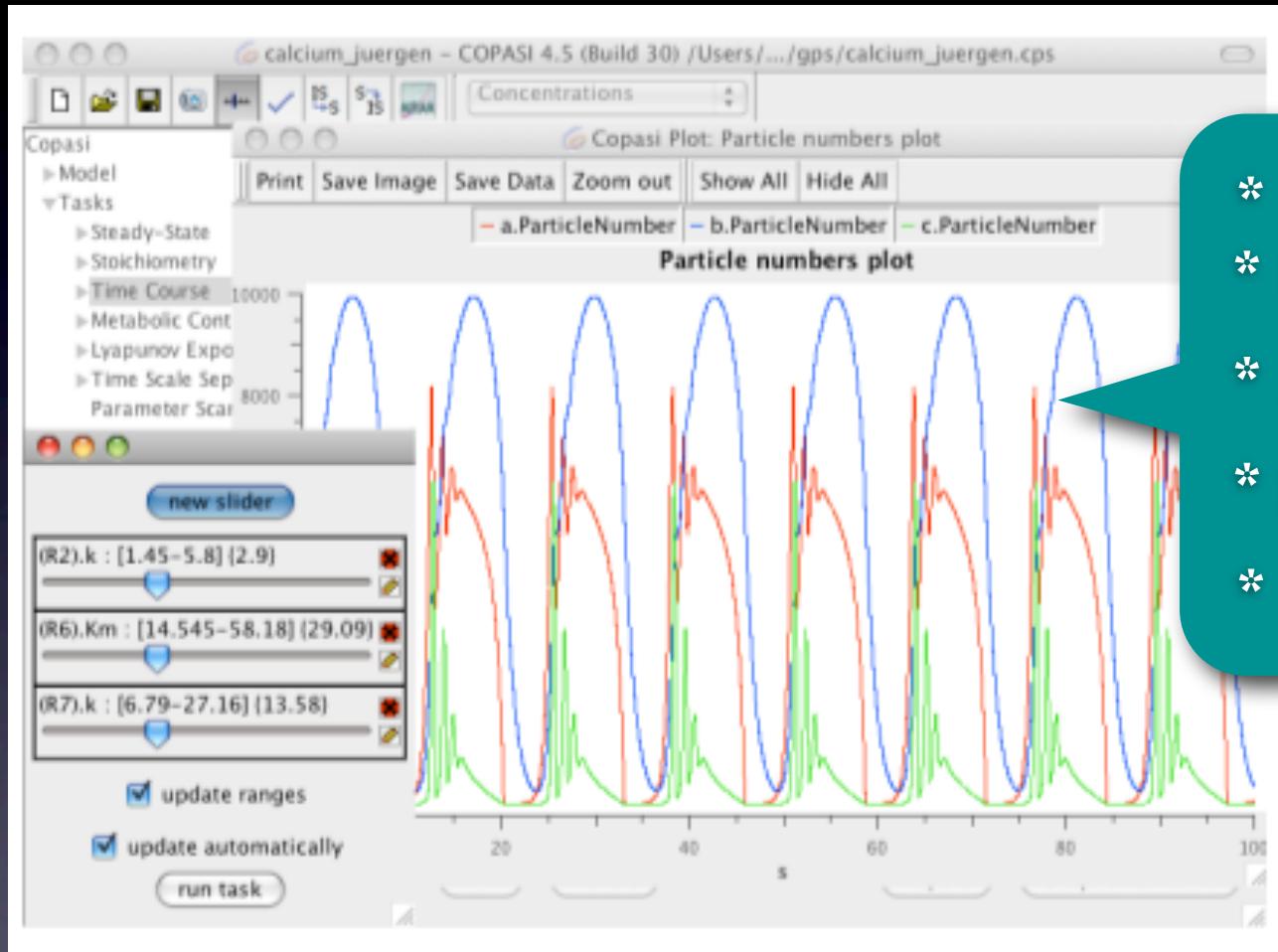
- SBML Level 3 version 1 tests
- 980 SBML tests (passed 100%)

http://sbml.org/Facilities/Online_SBML_Test_Suite



シミュレータの紹介

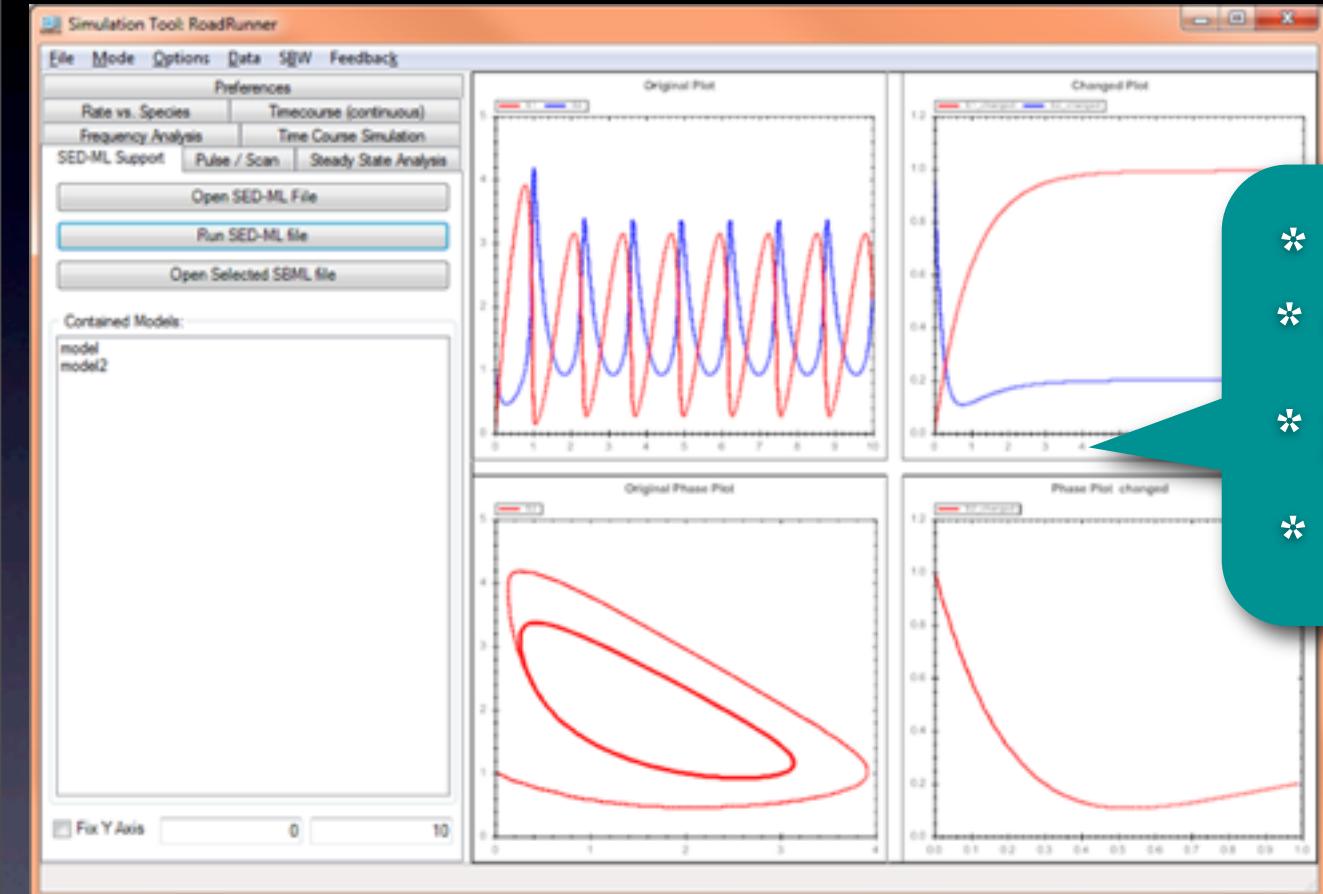
COPASI



- * ODE, SSA
- * LSODA (陰解法もOK)
- * パラメータフィッティング
- * 解析
- * C++

Univ. of Manchester, Univ. of Heidelberg, VBI
<http://copasi.org/>

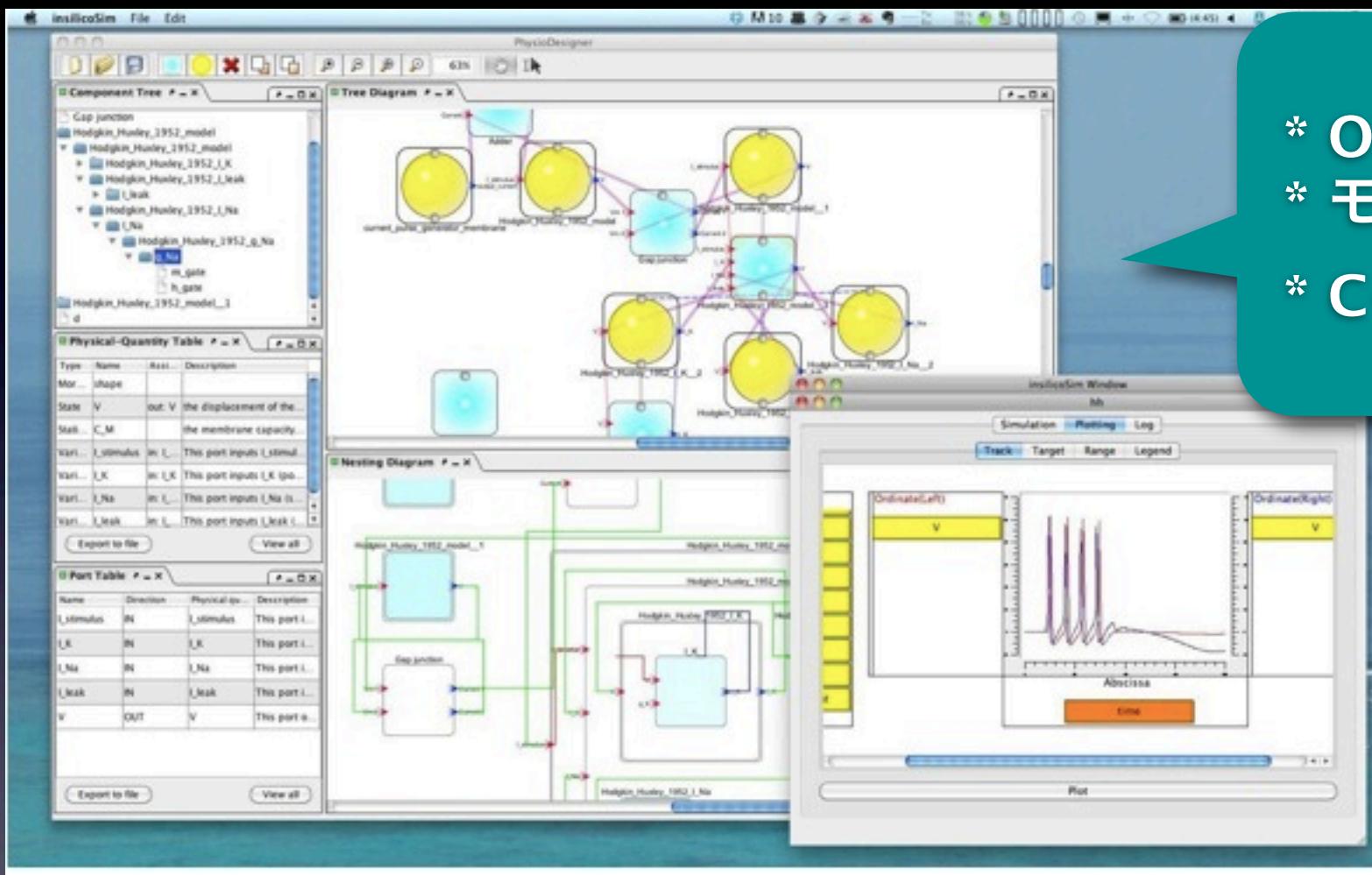
RoadRunner



- * ODE
- * CVODE (陰解法もOK)
- * 解析ツールと連動
- * C# (.NET, Mono)

Univ. of Washington, Caltech
<http://sf.net/projects/roadrunner>

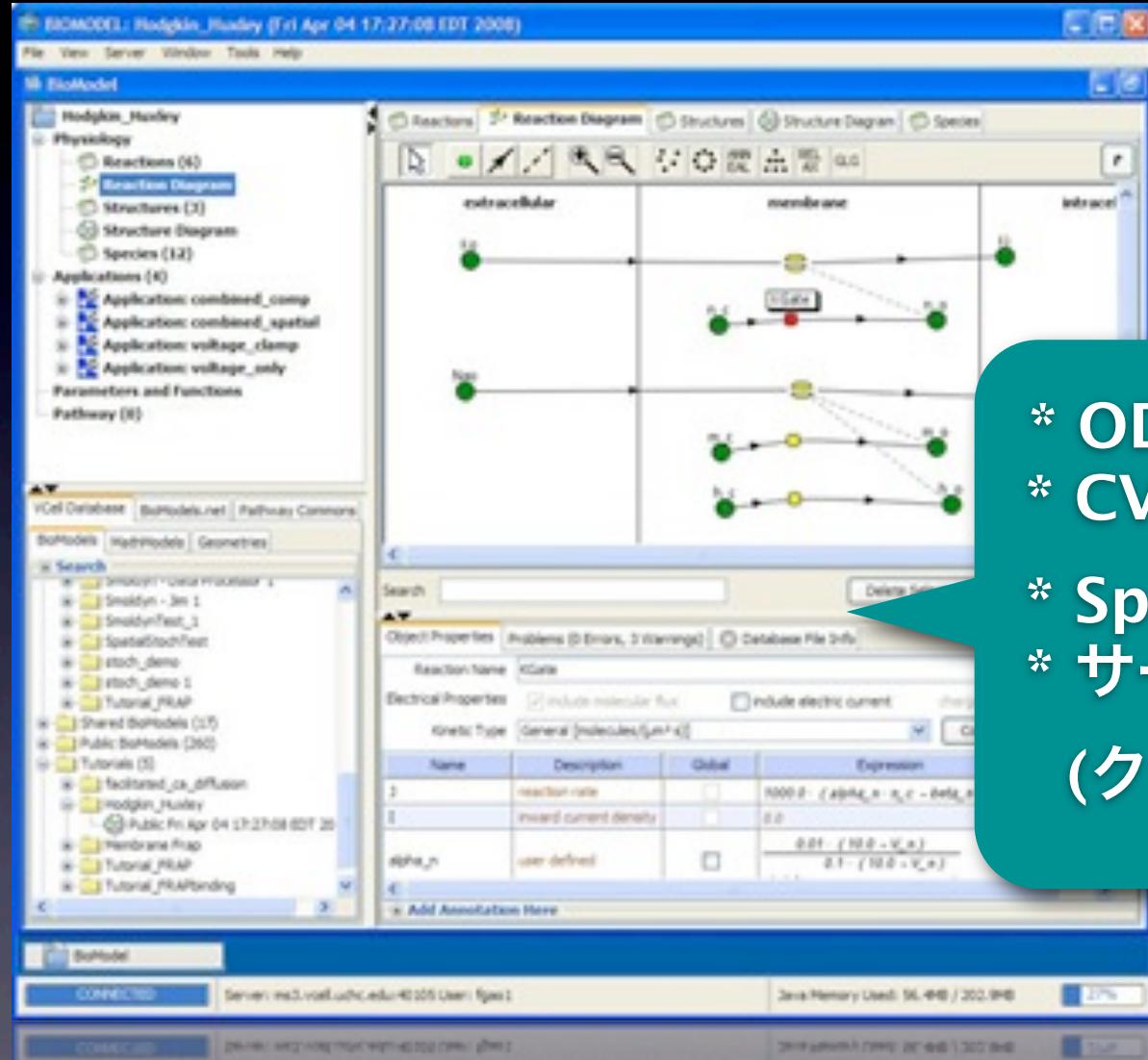
PhysioDesigner



Osaka University, OIST

<http://physiodesigner.org/>

VCell



- * ODE, SSA ,PDE, DAE
- * CVODE/IDA (陰解法もOK)
- * Spatial Stochastic (Smoldyn)
- * サーバサイド
(クライアント版もリリース)

University of Connecticut Health Center
<http://vcell.org/>

CellDesigner

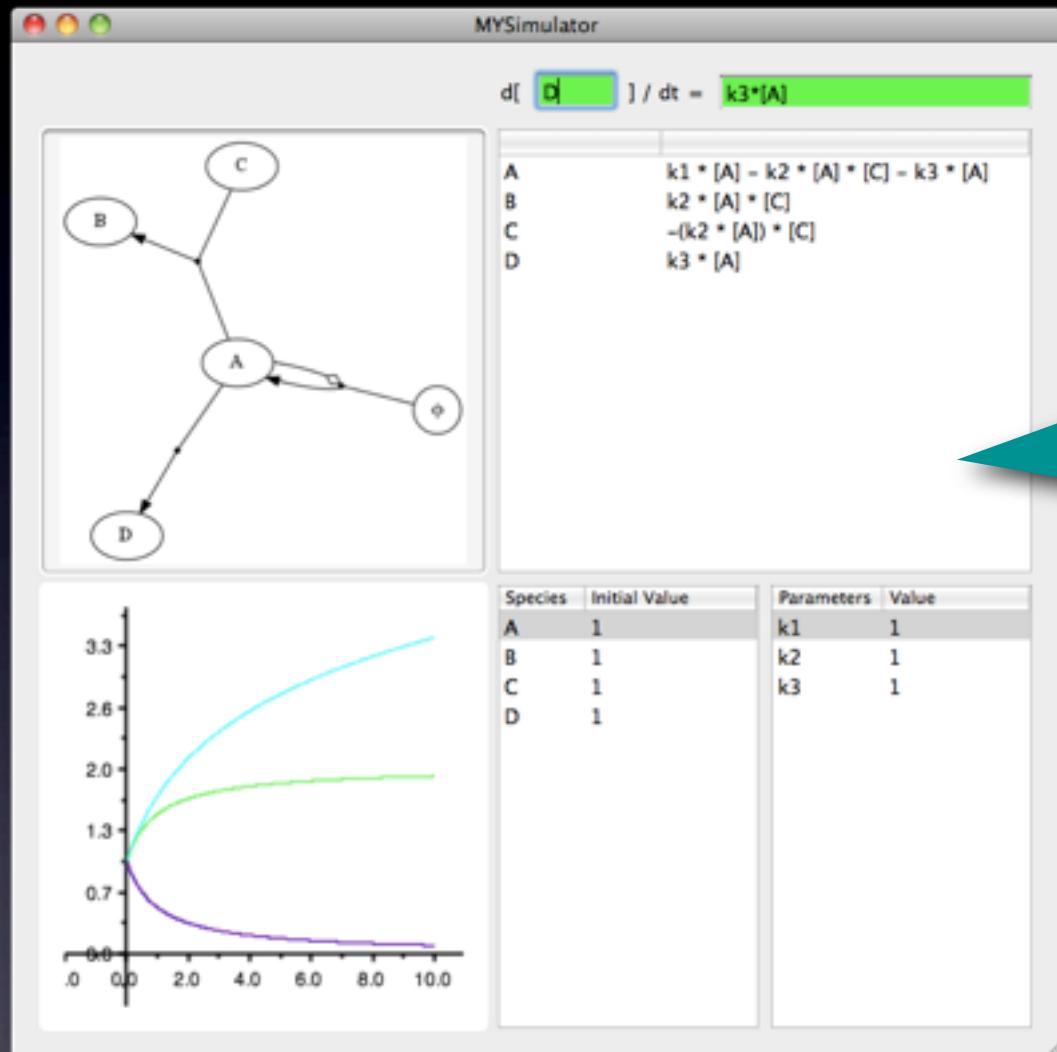
The screenshot shows the CellDesigner 4.2 interface. On the left, a block diagram of a MAPK signaling pathway is displayed, consisting of four protein species (MKKK, MKK, MAPK) and their interactions via seven reactions (J0-J6). Below the diagram is a table of species and their initial quantities. In the center, a simulation graph plots concentration over time for various species, with a legend on the right. A callout bubble on the right side lists features of the software.

Species	id	name	speciesType	compart...	initialQuantity	positio...	include...
PROTEIN	MKK	MKK	uVol	280.0	inside		
PROTEIN	MKKK	MKKK	uVol	90.0	inside		
PROTEIN	MAPK	MAPK	uVol	280.0	inside		
PROTEIN	MK..._1	MKKK	uVol	10.0	inside		
PROTEIN	MK..._2	MKK	uVol	10.0	inside		

* ODE, SSA(COPASI)
* CVODE (陰解法もOK)
* パラメータスキャン
* C, C++, Java

JST, SBI, Keio University
<http://celldesigner.org/>

最速モデル構築



- * ODE
- * 陰解法もOK
- * 微分方程式 → ネットワーク
- * C++, Objective-C

Keio University
Coming soon!

数式処理

● Maxima, wxMaxima

● Wolfram Alpha

```
(%i4) 'diff(y(x),x,2) + 'diff(y(x),x) + y(x) = 0;
(%o4)  $\frac{d^2}{dx^2}y(x) + \frac{dy}{dx}y(x) + y(x) = 0$ 

(%i5) desolve('diff(y(x),x,2) + 'diff(y(x),x) + y(x) = 0, y(x));
(%o5) y(x) = \frac{x}{\sqrt{3}} \left( \sin\left(\frac{\sqrt{3}x}{2}\right) \left( 2 \left( \frac{dy}{dx}y(x) \Big|_{x=0} + y(0) \right) - y(0) \right) + y(0) \cos\left(\frac{\sqrt{3}x}{2}\right) \right)
```

```
(%i6) x^k;
(%o6) x^k

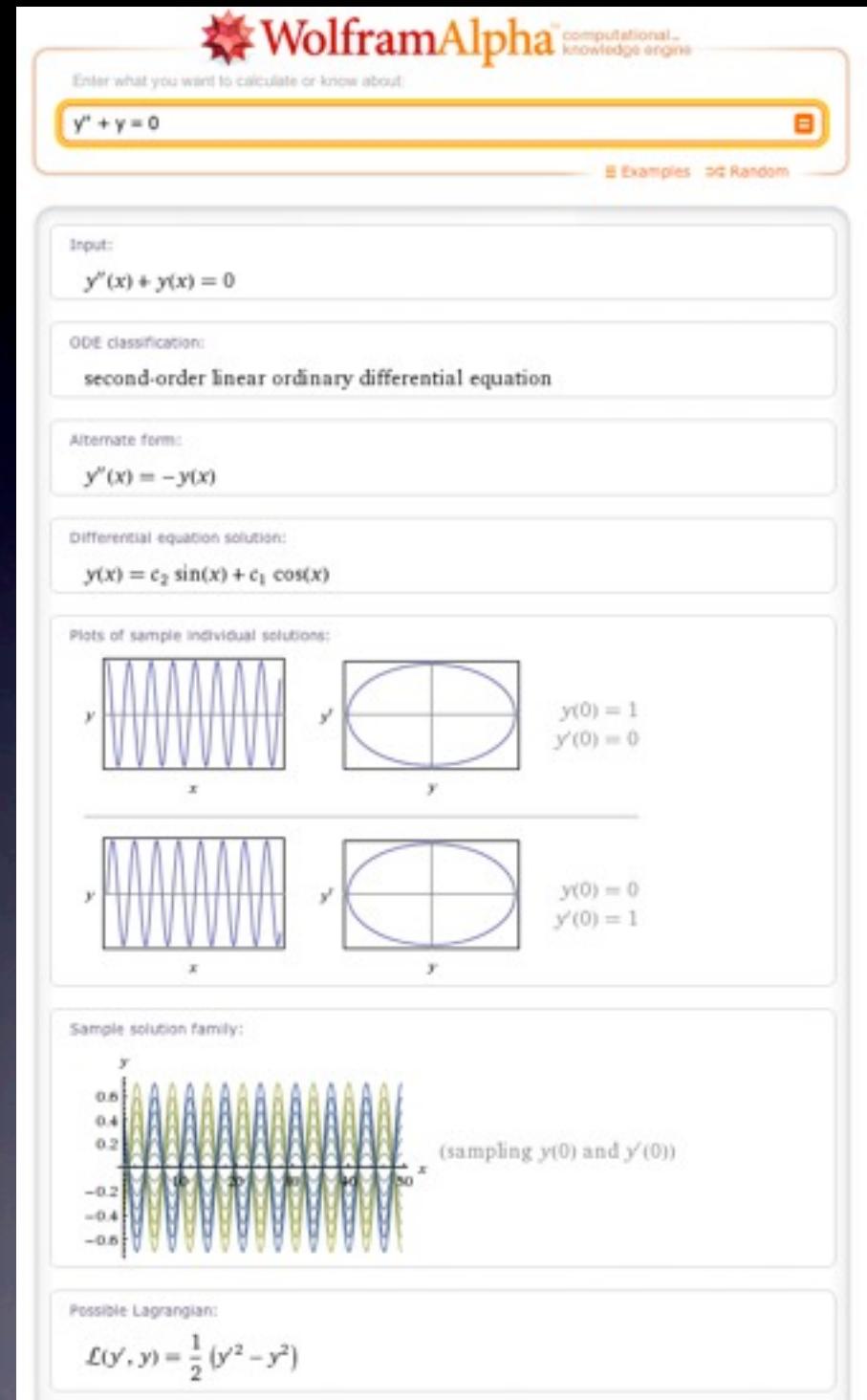
(%i7) diff(x^k, x);
(%o7) k x^{k-1}

(%i8) diff(x^k, x, 2);
(%o8) (k-1) k x^{k-2}

(%i9) integrate(x^k, x);
Is k+1 zero or nonzero? n;
(%o9)  $\frac{x^{k+1}}{k+1}$ 

(%i10) integrate(sin(x), x);
(%o10) -cos(x)
```

wxMaximaへようこそ
Zoom set to 130%



謝辞

- 東京大学 生産技術研究所 小林徹也 淄教授
- 慶應義塾大学 理工学部 広井賀子 助教

松井達広
偏微分方程式

中村和成
陰解法
高速化(GPU)

瀧沢大夢
陰解法、イベント処理、
遅延・代数微分方程式

田平章人
構文解析、陰解法
高速化(CPU)

